Experimental evidence of omnidirectional elastic bandgap in finite one-dimensional phononic systems

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We present an experimental demonstration of the occurence of omnidirectional reflection in a finite multilayer of two elastic materials. This property is demonstrated in three different samples consisting of a few periods of Pb/Epoxy. The effect is fullfiled thanks to the large mismatch between the elastic parameters of the two materials in the multilayer. The thicknesses of the layers were chosen in order to have the omnidirectional gap at a few hundreds of kHz. The theoretical treatment of the phononic band structure of the corresponding superlattice as well as the transmission properties through the finite structures are in good agreement with the transmission measurements. © 2004 American Institute of Physics. [DOI: 10.1063/1.1766074]

In the field of photonic crystals, it has been shown that omnidirectional reflection can take place even in the absence of a complete bandgap. Thus, finite superlattices of two different dielectric materials exhibit that property for any incident angle and polarization if the impinging light travels in a medium with relatively low index of refraction, like air.[1,2] The existing analogy between the scattering properties of electromagnetic waves and the scattering of elastic waves suggests that this type of features should appear also in one-dimensional (1D) phononic crystals. In fact, the phenomenon of omnidirectional reflection has been theoretically discussed for several 1D phononic structures.[3–6]

This letter presents the experimental demonstration of an omnidirectional elastic bandgap in the field of 1D phononic structures. For that purpose, we have extended the theoretical analysis and provided the construction of a multilayer (ML) omnidirectional reflector workable in ultrasound frequencies. The structure is made of thin layers of two alternating materials with different elastic constants (lead and epoxy) bounded on both sides by nylon, which acts as the propagating medium. This structure is chosen because of its easy fabrication in a standard laboratory.

First, the general features of the propagation properties of the finite structure have been theoretically analyzed from the phononic band structure of the corresponding infinite structure. The numerical procedure employed to get the phononic bands of a Pb/epoxy infinite superlattice is similar to the one described in Ref. 7. All of the interfaces are taken to be parallel to the (x,y) plane. Since the materials are isotropic elastic media, the transverse elastic waves, polarized perpendicular to the sagittal plane (x,z), are uncoupled from

sagittal waves polarized within this plane, for any value of the propagation vector k_x, (the wave vector parallel to the surface). Table I reports the elastic parameters employed in the calculations. The layer thicknesses d_1 and d_2 of Pb and epoxy, respectively, were chosen so that the structure has its omnidirectional gap in the working frequencies of the transducers. We used $d_1=d_2=1$ mm to generate a gap centered at around 300 kHz.

It is convenient to display the solutions of the infinite structure by projecting the band structure $\omega(k_x,k_z)$ onto the $\omega-k_x$ plane. Figure 1 shows the projected band structure for transverse and sagittal acoustic waves. For every value of k_x, the shadowed and white areas corresponds to the regions where the propagation of waves are allowed or forbidden, respectively. Due to the large contrast between the elastic parameters of Pb and epoxy, the gaps are rather large. It is noticeable in Fig. 1 how this superlattice does not display any complete band gap, a feature that it is general for any 1D phononic crystal. However, we are interested in longitudinal waves originating from the nylon, which is the homogeneous medium external to the periodic structure. These waves satisfy the condition $\omega \geq c_{l,Ny}k_x$. Therefore, the only states in Fig. 1 that are lying in the shadowed area above the straight lines $L_{l,Ny}=c_{l,Ny}k_x$ can propagate in both the nylon and the superlattice. So, the frequencies in between the horizontal dashed lines in Fig. 1 accomplish the criterion for omnidi-

TABLE I. Physical parameters of the materials employed in the sample. C_l and c_t are the longitudinal and transversal velocities of the material, respectively, and ρ is the density.

Material	ρ(kg m^{-3})	c_l(m s^{-1})	c_t(m s^{-1})
Pb	10 760	1960	850
Epoxy	1140	2770	1300
Nylon	1110	2600	1100

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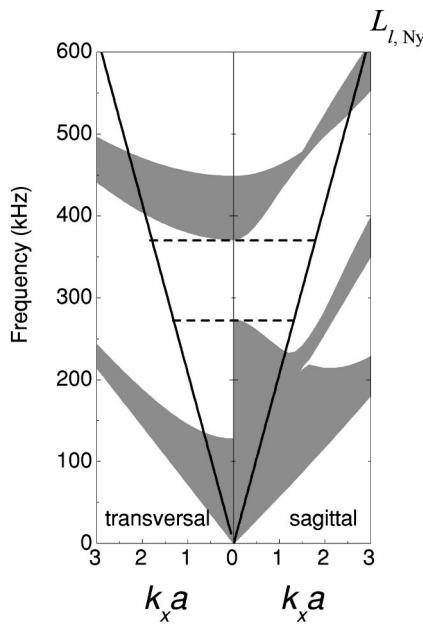
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FIG. 1. Projected band structure of sagittal and transverse elastic waves in a Pb/epoxy superlattice with layer thickness $d_1=d_2=1$ mm. The frequencies (in kHz) are represented in function of the reduced parallel wave vector $k_x a$, where $a=d_1+d_2$. The horizontal dashed lines enclosed the frequencies of no transmission at every angle; i.e., the band of omnidirectional reflection.
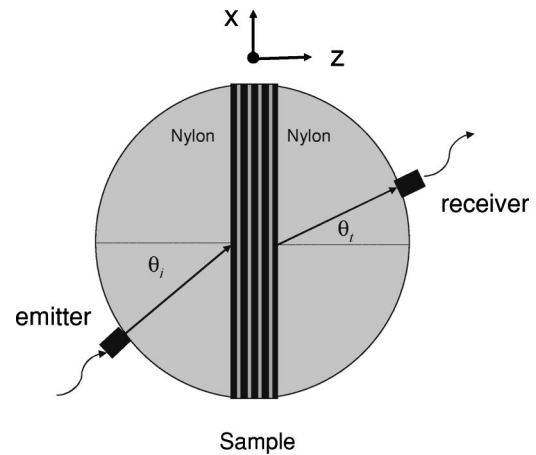

FIG. 2. Schematic drawing of the experimental setup. The ML sample is placed in between two semicircles of nylon that acts as the propagating medium. θ_i defines the angle of incidence and θ_t is the angle of transmission.

rectional reflectivity, there are no states of the multilayer structure inside the sound propagating cone. Our calculation predicts such omnidirectional band gap in the range of 273 kHz $\leqslant \omega/2\pi \leqslant$ 371 kHz, which corresponds to the normal incidence band edges of the sagittal modes. At this point, let us remark that our proposed structure does not have the property of omnidirectional reflection for transversal waves, for which the velocity is about half of the longitudinal waves (see Table I).

To characterize the omnidirectional band gap, we have studied the transmission amplitude for longitudinal waves crossing finite ML structures. Transmission measurements have been performed in three different ML samples that were constructed by alternating layers of Pb and epoxy, which are glued togheter. All of the samples are 16 mm thick. Sample No. 1 contains two periods of Pb/epoxy plus and additional layer of Pb; i.e., three Pb layers in total. Sample Nos. 2 and 3 have four and six Pb layers, respectively. All of the samples are symmetric with respect to their center layer and are sandwiched between two semicircular pieces of nylon (the substrate) to perform the measurements. A film of medical gel for ultrasounds is used to produce a better contact between the samples and the nylon. The experimental setup is described in Fig. 2. In brief, a transducer, the emitter, is employed to excite a longitudinal wave in the substrate. The wave crosses the ML sample and is recorded by a second transducer, the receiver, placed in contact with the substrate at the opposite side. Also, the medical gel is employed to contact substrate and transducers. The semicircular shape of the substrate (radius 15 cm and thickness 5 cm) allows one to explore different angles in emission as well as in reception. Since the contact area of emitter (receiver) transducer has a radius of 4 cm (3 cm) the measurements in angles have an error of about ±5°. The central frequency of both transducers is 500 kHz, their bandwidth being approximately 50%. A ultrasonic square wave pulser/receiver, model

5077-PR Panametrics, is used to excite (to amplify) the signal emitted (received) by the emitter (receiver) transducer. The piezoelectric is excited at 400 V. The signal generated by the incident vibration is recorded by the receiver and digitalized by a digital oscilloscope connected to a computer to store the data.

Our experimental setup only involved the propagation of vibrations in the sagittal $x–z$ plane. On the one hand, the emitter excites a longitudinal wave that after the mode conversion inside the multilayer becomes two transmitted waves, one of longitudinal character and the other of transverse character. On the other hand, the receiver detects vibrations normal to the contact interface; i.e., along the radial direction. In other words, the vibrations detected consist of the projection in the radial direction of the transmitted waves (longitudinal and transverse). We analyzed nine incident directions θ_i (0°, ±20°, ±40°, ±60°, and ±80°). For each angle θ_i we recorded the transmission at nine different angles of transmission θ_t (0°, ±20°, ±40°, ±60°, ±80°). Figure 3 shows the transmission amplitude measured at different angles of incidence for the three samples analyzed. At angles $\theta_i \neq 0°$, it is expected that vibrations should be excited in the regions of frequencies where the bulk sagittal bands overlap with the incident line L_{θ_i}, which is determined by the equation $L_{\theta_i} = c_{l,Ny} k_x / \sin \theta_i$. Each spectrum corresponds to the sound recorded by the receiver at the specular transmitted direction. For directions θ_t different from the specular only noise is measured resulting from the waves originated by multiple reflections on the substrate's borders. Notice that the transmission is almost negligible for the three samples in the regions where a gap (shadowed regions) is predicted by the band structure. For the thinner samples, Nos. 1 and 2, a residual transmission is still observed that fully disappears in the thicker sample. Besides, the regions of no transmission overlap for this sample defining its omnidirectional band gap, which is in very good agreement with the theoretical predictions. This result indicates that three periods of Pb/epoxy are enough to achieved the property of omnidirectional reflection.

The theoretical transmission spectra are also obtained by applying the boundary conditions at the different interfaces forming the ML and solving the resulting system of equations. The radial components of the transmitted sagittal
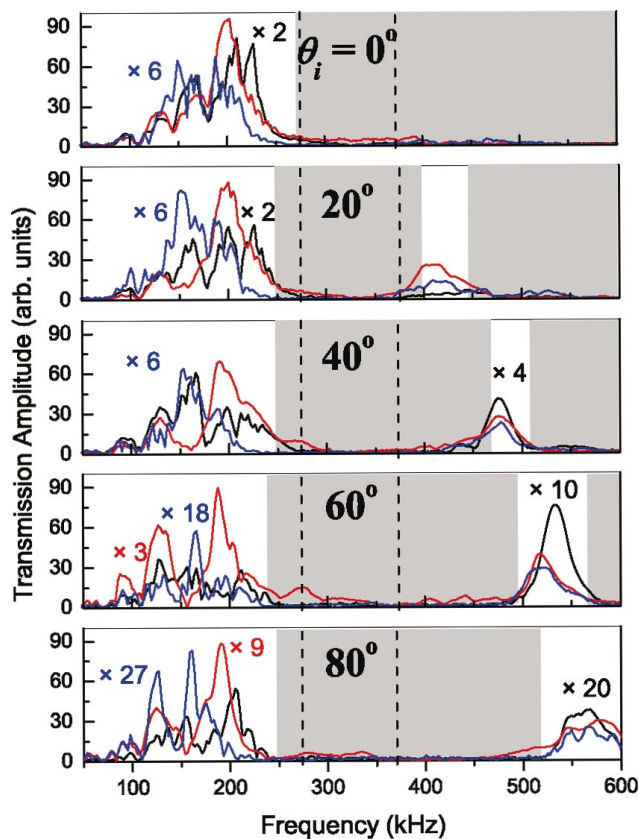
FIG. 3. (Color) Transmission amplitude measured as a function of frequency for longitudinal waves at normal, 20°, 40°, 60°, and 80° angles of incidence. The red lines display the spectra for Sample No. 1, the black and blue lines correspond to Samples 2 and 3, respectively. Each spectrum is represented with a given factor. The shadowed areas define the gaps calculated along each angle of incidence θ_i, and the vertical dashed lines enclosed the omnidirectional gap (see Fig. 1).
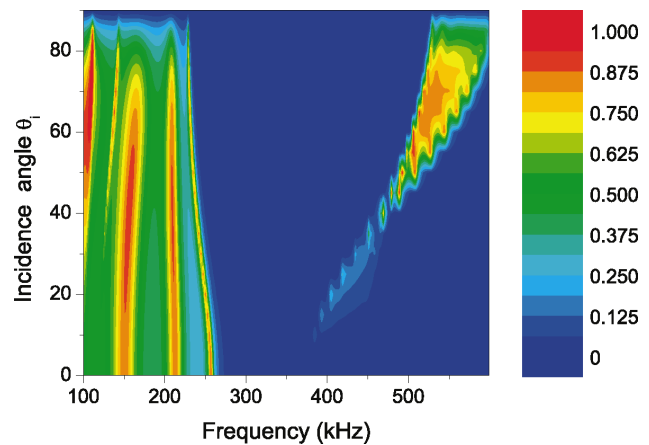

FIG. 4. (Color) Theoretical transmission spectra of longitudinal waves across a symmetric ML sample consisting of four periods of Pb/epoxy plus a final Pb layer. The modulus of the total radial amplitude is represented as a function of the incident angle in the frequency region of interest. The blue (red) color define minimum (maximum) amplitude (see the inset). A omnidirectional band gap is clearly observed between 260 kHz and 380 kHz.

waves (transversal as well as longitudinal) are taken into account to calculate the modulus of the transmission amplitude. Figure 4 shows the transmission calculated for a ML structure consisting of four periods of Pb/epoxy plus a final layer of Pb. A good agreement between the region of zero transmission (blue region in Fig. 4) and experimental measurements in Fig. 3 is found. Also, it must be noticed how the dispersive transmission band between 400 and 600 kHz is well reproduced in the data of Fig. 3. Then, the correspondence between the frequencies with omnidirectional zero transmission in finite structures and the omnidirectional band gap predicted by the photonic band calculation (see Fig. 1) has been shown.

In summary, an experimental demonstration of the occurence of an omidirection band gap for longitudinal waves impinging on an elastic multilayer has been reported. Nevertheless, let us point out that in order to achieve a complete omnidirectional reflector (i.e., also having omnidirectional band gaps for transversal waves) a ML structure must be designed.

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