Extensions to "Virtual Reference Feedback Tuning: a direct method for the design of feedback controllers"

Antonio Sala, Alicia Esparza

Dept. of Systems Engineering and Control, Universidad Politécnica de Valencia; Camino de Vera, 14; E-46022 Valencia, Spain. Email: {asala,alespei}@isa.upv.es

Abstract

The papers (Campi *et al.*, 2002; Campi *et al.*, 2003) present a direct controller synthesis procedure that uses identification algorithms applied to filtered input-output plant data. This contribution discusses variations that, in some cases, may alleviate noise-induced correlation (in the open-loop case) and allow the applicability of the approach to unstable plants. Importantly, it also introduces an invalidation test step based on the available data (*i.e.*, prior to experimental controller testing), to check if the flexibility of the controller parameterisation and the approximations involved are suitable for the design objectives or, on the contrary, the resulting closed loop may be unstable.

Key words: system identification, controller tuning, adaptive control, data-based controller design

1 Introduction

The papers (Campi *et al.*, 2002; Campi *et al.*, 2003) present a controller synthesis procedure via identification (ID) techniques, directly using plant input-output data without resorting to intermediate process models. The procedure was named "*virtual reference feedback tuning*" (VRFT). This short contribution discusses simple alternatives that, in some cases, may extend the applicability of the ideas in their algorithm [1]. The interested reader is also referred to (Safonov and Cabral, 2001) and references therein where a related approach is described. The virtual reference approach can also be used for controller *invalidation* in supervision or adaptation tasks (Mosca and Agnoloni, 2001; Safonov and Tsao, 1997). This idea will also be addressed in this contribution. Controller ID can also be used as a controller order reduction tool (Landau *et al.*, 2001).

The structure of the contribution is based on a single discussion section, followed by examples pointing out the applicability of the presented ideas. Later, some summarising conclusions are drawn.

[1] In the following, reference to figures and equations in (Campi *et al.*, 2002) will be denoted by prefixing with "CLS", for instance "Figure CLS1".

2 Discussion

Preliminaries and notation. (Campi *et al.*, 2002) identify a controller from input-output data (u, y) gathered from a process P, given a target closed-loop behaviour M to be attained when subject to a setpoint input r. The closed loop is depicted in figure CLS1, where $u = C(r - y)$, with a parameterised controller $C(\theta)$, $\theta \in \mathbb{R}^n$. C_0 will denote the "ideal" controller achieving the tracking behaviour M, *i.e.*, the one with transfer function $C_0 = \frac{M}{(1-M)P}$. Depending on the parameterisation of $C(\theta)$, C_0 may not belong to the controller set $\mathcal{C} = \{C(\theta)|\theta \in \mathbb{R}^n\}$. The proposed controller is the one minimising cost index (CLS6), that can also be written as:

$$J_{MR}(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|W|^2|P|^{-2}|C^{-1}(\theta) - C_0^{-1}|^2}{|(PC)^{-1}+1|^2|(PC_0)^{-1}+1|^2}\,d\omega \quad (1)$$

where W is a user-defined frequency weight. The procedure evaluates a candidate controller by generating a *virtual reference*: $r(\theta) = C^{-1}(\theta)u + y$, trying to adjust it to fit a target behaviour $\bar{r} = M^{-1}y$. We will denote as *virtual tracking error* the quantity: $e = (M^{-1} - 1)y$. Ideally, "perfect" control would be achieved if a parameter value θ^* were found so that $C(\theta^*)e = u$, *i.e.*, $C(\theta^*) = C_0$. In (Campi *et al.*, 2002), it is shown that (CLS6) can be approximately minimised by posing an output-error (OE) setup with the cost index (CLS2):

$$J_{VR}^N = \frac{1}{N}\sum_{i=1}^{N}(u_L^i - C(\theta)e_L^i)^2 = \|L(u - C(\theta)e)\|_{2,N} \quad (2)$$

where u_L and e_L denote sequences, with N data points, obtained by filtering with a suitable prefilter, L, the input and virtual error sequences. The notation $\|\cdot\|_{2,N}$ has been introduced to denote the finite average of squares. The ID setup can be formulated directly on input-output data (Campi *et al.*, 2003), as $e = (M^{-1} - 1)y$. The setup proposed in (Campi *et al.*, 2002) involved a prefilter $L = M(1-M)T_u^{-1}W$, where T_u is a filter such that $|T_u|^2 = \Phi_u$ (Φ_u is the power spectral density of $u(t)$), resulting in:

$$J_{VR}^N = \|WM(1-M)T_u^{-1}u - C(\theta)W(1-M)^2T_u^{-1}y\|_{2,N} \quad (3)$$

In (Campi *et al.*, 2003), analogous conditions to set up an ID experiment for the input sensitivity $C(1+PC)^{-1}$ to approach a target U are discussed. Then, by selecting different frequency weights for each of the criteria, a sort of "mixed sensitivity" approach is proposed.

Let us put forward some remarks to the procedures in (Campi *et al.*, 2002) that enhance their applicability.

Remark 1 (full parameterisation): In (Campi *et al.*, 2002) and (Campi *et al.*, 2003) linearly parameterised controllers $C(\theta) = \beta^T(z)\theta$ are used, allowing for one-shot least-squares formulae. However, fully parameterised controllers $C(\theta) = (\theta_{10} + \theta_{11}z^{-1} + \dots)/(1 + \theta_{21}z^{-1} + \dots)$ can be identified (by iterative optimisation) with widely available algorithms, such as those in Matlab System ID Toolbox. Notwithstanding, pole-zero cancellation issues appear with unstable and non-minimum phase plants, to be later discussed.

Remark 2 (open-loop control): If the virtual error e in (2) is replaced by the target reference $M^{-1}y$, a feedforward controller (for a stable plant) can be synthesised by direct ID. Furthermore, validation issues discussed in Section 2.1 are straightforward, as stability of the identified controller is the only requisite.

Remark 3 (controller inverse): An alternative setup to the one in (Campi *et al.*, 2002) may be identifying the controller inverse, using a parameterisation of $C^{-1}(\theta)$:

$$J_{IVR}^N = \|C^{-1}(\theta)Lu - L(M^{-1} - 1)y\|_{2,N} \quad (4)$$

with a sensible choice of L. If the output of the process is corrupted by additive noise uncorrelated with the input u, OE algorithms will provide an unbiased estimate of θ, contrary to (2) where instrumental variables are needed. Other advantages and disadvantages of this setting with respect to the original one in (Campi *et al.*, 2002) are outlined after Remark 4.

As $Le = L(M^{-1} - 1)y = L(M^{-1} - 1)Pu = LC_0^{-1}u$, the minimisation of (4), asymptotically (N tending to infinity) is equivalent to minimising:

$$J_{IVR} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |C^{-1}(\theta) - C_0^{-1}|^2|L|^2\Phi_u \, d\omega \quad (5)$$

As the original cost index (CLS4) can be written as (1), following a line of reasoning parallel to the one yielding (CLS10) (*i.e.*, replacing C by C_0 in the denominator of (1)) the suggested L would be:

$$|L|^2 = |W|^2|M|^4|P|^{-2}\Phi_u^{-1} = |W|^2|M|^4\Phi_y^{-1} \quad (6)$$

Note that a sort of a prewhitening filter $T_y(z)$ ($|T_y|^2 = \Phi_y$) needs to be identified, or approximately replaced by a high-pass filter. Inserting the above L in the original equation (4), the result can be expressed as

$$J_{IVR}^N = \|C^{-1}(\theta)WM^2T_y^{-1}u + WM(1-M)T_y^{-1}y\|_{2,N} \quad (7)$$

Remark 4 (unstable poles and zeros): In the last paragraph of the example in (Campi *et al.*, 2002), the authors say that the unstable zero "does not tend to be cancelled by the controller", evidently, as the controller denominator is fixed. With unstable or non-minimum-phase plants, that may no longer be the case if a more general parameterisation, as discussed in Remark 1, is used. If the parameterisation is flexible enough, $C^{-1}(\theta)$ will tend to mimic the plant dynamics (shown by replacing y by Pu in (4)); analysing in the same way (3), $C(\theta)$ will tend to cancel the plant dynamics. To address this issue, there are three alternatives: (a) changing the parameterisation: fixing some parameters, as in the original reference (Campi *et al.*, 2002), using reduced-order controllers, *etc.* (b) devising M so that the unstable or non-minimum-phase factors of P are cancelled in (2) or (4), at least approximately, and (c) using OE identification routines either in (2) or in (4) as a sole option, to obtain stable controllers with non-minimum-phase plants or to achieve stable controller inverses with unstable plants, respectively.

So, the possible advantages of ID of $C^{-1}(\theta)$ are:

(1) Tolerance to additive noise at the plant output y (with OE model structure and open-loop data),
(2) If data come from unstable plants (stabilised by a prior low-performing controller), an output-error ID algorithm will not cancel the unstable poles (whose dynamics are present in u).

On the other hand, disadvantages of the $C^{-1}(\theta)$ approach might be:

(1) If data come from a non-minimum-phase plant, free parameters in the numerator of $C^{-1}(\theta)$, estimated

via the VRFT approach, open the possibility of identifying the non-minimum-phase zeros (whose dynamics are present in y) as controller poles.

(2) For results approximately coincident to those in (Campi *et al.*, 2002), a prewhitening T_y needs to be identified. This may be considered dual to the need of identifying instrumental variables (such as a high-order plant model) in (Campi *et al.*, 2002) for robustness to additive output noise.

2.1 Invalidation test

The "tricky" issues of the approach here and in (Campi *et al.*, 2002) are to choose a flexible enough controller structure, a suitable target model M and a weight W to start with. Indeed, the presented procedures may fail (regarding stability or performance of the resulting loop), due to the following reasons:

First, the resulting identified controller may yield an unstable loop if widely-known interpolation constraints are not fulfilled (Remark 4). For that purpose, the calculation of a target behaviour (and a first stabilising controller with unstable plants) with an approximate plant model seems to be a sensible approach.

Second, the target high-frequency roll-off of M must be high enough to bias the controller towards a limited high-frequency activity (at least in a stable, minimum-phase case) in order to achieve reasonable robustness margins and measurement noise filtering.

Last of all, frequency weights when $C_0 \notin \mathcal{C}$ and the various approximations involved (finite number of samples, prewhitening, small controller order, disturbance-induced bias and variance errors in the ID procedure, differences between J_{MR} and J_{VR}^N or J_{IVR}^N, etc.) also influence the final result.

Additional issues may arise in the "mixed sensitivity" approach in (Campi *et al.*, 2003), where the search is not made over the set of stabilising controllers as in standard $\mathcal{H}_2$ designs, so the resulting controller with "optimal" fit might be a non-stabilising one.

As a conclusion, an important issue is devising how to use the available data to check (prior to actual closed-loop tests) whether the obtained controller may yield a stable loop or, on the contrary, an unstable loop may be likely. To that purpose, an identification-based invalidation test is suggested. Indeed, the most evident test would be identifying a model of the plant and checking if the obtained controller yields a stable closed loop with it. An alternative approach is to directly identify some of the "virtual" closed-loop transfer functions. For instance, the procedure below is suggested:

(1) Generate the virtual reference $r = C^{-1}(\theta)u + y$,

(2) Identify a model of the transfer function H between [2] u and r: $r = H(\theta')u$.

(3) Check if the characteristics of the identified H are satisfactory. In particular, $H^{-1} = u(z)/r(z)$ approximates the input sensitivity $F = C(1+CP)^{-1}$, hence it should be stable if the final loop is to be internally stable: if H has unstable zeros, the controller is invalidated and the procedure must be restarted with different design parameters (M, W, controller order, *etc.*).

The procedure in a general case is not as straightforward as it might seem, as issues regarding disturbance-induced bias (instrumental variables), model structure and order, etc. must be sorted out to obtain meaningful results. For instance, depending on the pole-zero characteristics of the actual H, different model structures should be used for ID: in unstable plants F is non-minimum-phase, so $F^{-1} \approx H$ is unstable and cannot be accurately identified by, say, an OE structure (other ones such as ARX, ARMAX are advised).

3 Examples

Flexible link. A flexible transmission system benchmark is used in (Campi *et al.*, 2002) to demonstrate the ideas in that paper. For the sake of comparison, the same plant, M and W were used when identifying the controller inverse with a fixed numerator $1-z^{-1}$ and five denominator parameters, *i.e.*, the same class of controllers proposed in (Campi *et al.*, 2002). The OE algorithm in Matlab System ID Toolbox gave result to the closed-loop step responses in Figure 1. Prewhitening was carried out via ID of a 6th order auto-regressive model for y. The results are similar to those from (Campi *et al.*, 2002), noting that controllers in plots `W`, `NW` have been directly identified from noisy open-loop data (white noise additive disturbance at output, signal/noise ratio=10, as in the referred paper). Regarding the invalidation test, identifying $H(\theta')$ with a 6th-order ARX structure, the results were satisfactory (H was minimum-phase).

If a full 5th-order controller inverse (parameterised numerator and denominator) is identified, the controller cancels the unstable zero of the transmission system. Fortunately, the identified H^{-1} (noisy data used) detects the situation, with a pole in $z = -1.81$, approximately coincident with the plant one (assumed unknown), invalidating the candidate C.

[2] As reference r, virtual error $r - y$, virtual target output Mr, etc. can be easily generated, other virtual closed loop functions may be identified. In the approach above, only the output data r is contaminated with output additive noise (with open-loop data), if u can be considered as a "clean" signal. ID of other transfer functions may have "contaminated" variables both at input and output (for instance, trying to estimate the achieved M from r and y), so severe biasing may occur unless special care is taken.

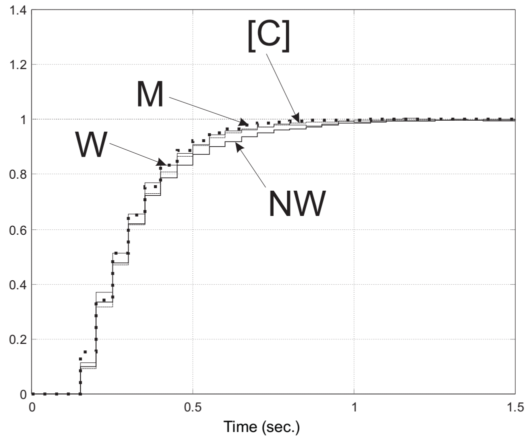

Fig. 1. Step responses. [C]: from Campi, M: target, W: with pre-whitening of y, NW: no prewhitening

A full 2nd-order controller $C = 0.4475(z^2 - 1.599z + 0.9555)/(z-1)/(z+0.03601)$ can be identified under the same settings. It passed the invalidation test and yielded an acceptable response (not shown).

Unstable system. A controller for the system $P(s) = 8/(1-2s)/(s+4)$, sampled at $T = 0.015$ s, is to be identified using an OE algorithm on C^{-1} following (7). A first stabilising controller $C_1 = -168(z-0.7)^2/(z-1.2)/(z+.8)$ is assumed available, for instance, designed on an approximate model. Data are gathered subjecting the closed-loop system to a white-noise additive input excitation (for simplicity, with no output disturbance). The target behaviour is set up as $M = 1 - S$ where the sensitivity function is $S = 1.2(z-1.025)(z-1)/(z-.95)^2$ containing a zero at $z = 1$ (zero steady-state error) and an unstable zero, trying to approximately match the corresponding pole of the process (lying at $z = 1.008$). With $W = 1$ and adding a fixed integrator to the controller, the identified controller:

$$C_2 = \frac{-76.7924(z^2 - 1.851z + 0.8609)}{(z-1)(z+0.9107)}$$

achieves the response in Figure 2. The validation procedure (6th-order ARX structure) also was satisfactory.

On the contrary, if a different $M = 0.022013(z + 16.74)(z-0.9682)(z-0.8028)/(z-0.93)^2/(z-0.5)$ (obtained by "playing" with the poles, zeros and high-frequency gain of a candidate sensitivity) is used under the same settings as before, the result is an unstable loop with the new controller. Fortunately, identification of H yields a pole of H^{-1} around $z = -1.04$, pinpointing instability with the data records available (prior to actually testing the controller).

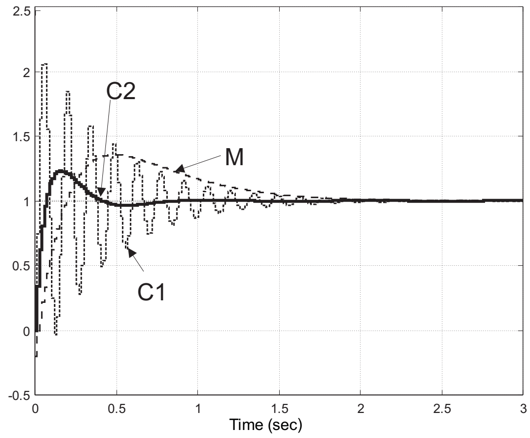

Fig. 2. $C2$: Step response of an unstable plant achieved via OE identification of $C_2^{-1}(\theta)$. $C1$: performance of initial controller for data gathering. M: target specification

4 Conclusions

Virtual reference feedback tuning approaches controller design via direct identification of the controller based on input-output data instead of resorting to an intermediate plant model. Both the controller and the controller inverse can be identified, in linear or full parameterisations. Each setting has specific advantages relating to additive output disturbances and unstable poles or zeroes: inverse controller output-error identification may be advantageous regarding output disturbances (open-loop) and unstable plants. Due to the series of approximations involved and/or the use of reduced-complexity controllers, an invalidation test is advised and, apart from obtaining a plant model, some identification-based possibilities are proposed using the virtual reference approach.

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