Communications Department

Analysis of the competition between operators in a femtocell scenario

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1 Objective

This paper analyses the effect of the entry of a femtocell operator in a mobile communications market where an macrocell operator exists.

The analysis is conducted using a game theory based model.

2 Problem statement

The problem is part of the implementation of the work programme financed by the *Ministerio de Economía y Competitividad* within the research project COHWAN. One of the project objectives is:

- To analyse through game theoretical modelling the economic aspects of the spectrum sharing. The analysis will depart from the framework provided by the cognitive radio networks ...

The problem stated was to answer the following question: which benefit do mobile communication users get from the entry of femtocell operators into the market?

The basic scenario comprises the following elements:

- Macrocell Operator (MO), which provide service to mobile communication users and which are entitled to use the spectrum for the provision
- Femtocell Operator (FO), which provide service to mobile communication users and which exploit an amount of spectrum owned by the Macrocell Operator. The latter lease spectrum to the former in order to improve the efficiency of the spectrum use and to increase their revenue.
- Users, which are the mobile communication users and which receive the service provided either by an MO or by an FO.

As regards the femtocell deployment, the following assumptions are made[1]:

- The femto base stations (femto-BSs) are deployed by the FO and no capital nor operational expenses are levied onto the users. The MO is not involved in the femto-BS deployment, either.
- The femto-BSs operate in open-access mode, as this mode allows the FO to offer service to users
- An orthogonal spectrum assignment is performed through MO-FO coordination, so that cross-tier interference is completely eliminated.

The concrete setting that will be analysed is shown in Fig. 1, where:

- One MO and one FO compete for the provision of service to the users. The MO and the FO are also referred to as *operator 1* and *operator 2*, respectively.
- No switching costs are incurred when users make the subscription decision

[1] For an introductory description of the deployment alternatives in femtocell networks, the reader may refer to Lin, Zhang, Chen, and Zhang (2011)

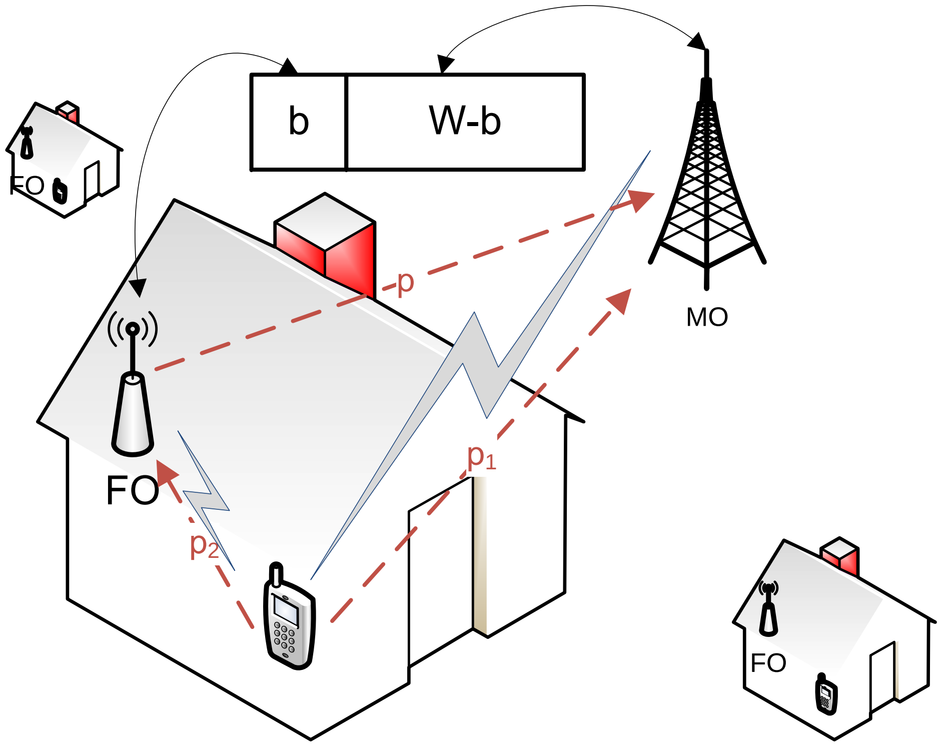

Figure 1: Scenario

- The FO have the same service coverage as the MO, and each user can access at every point in space to both MO and FO services. This assumption is also made by Duan and Huang (2011) and will be relaxed in future works.

The MO will lease an amount b of spectrum to the FO, keeping for itself the rest, up to W. The FO pays a price p m.u.[2] per unit of spectrum. Finally, a user would pay p_1 m.u. if she subscribed to MO service, or p_2 m.u. if she subscribed to FO service. All three prices p, p_1 and p_2 are referred to the same time period.

3 Literature review

There have been recent works on economic modelling of femtocell service provision:

- Yi, Zhang, Zhang, and Jiang (2012) covers a similar setting as in our current work, since it analyses the spectrum leasing between a macrocell service provider and a femtocell service provider. It is concerned not only with the amount of spectrum which is leased, but also with the amount of leased spectrum that the femtocell service provider is willing to share with the users of the macrocell service provider. However, Yi, Zhang, Zhang, and Jiang (2012) builds a non-standard model according to microeconomics, since they do not incorporate the

[2]m.u.= monetary units

users demand in the model; instead, they insert a proxy for the users utility in the service providers utility.

- Duan, Shou, and Huang (2011) models also a scenario where one macrocell operator leases spectrum to one femtocell operator, and it derives the conditions under which the former has incentives to lease. It assumes, as in our paper, that the femtocell operator does not incur costs and that it provides the same coverage as the macrocell operator, but it also examines the cases where each assumption does not hold. However, Duan, Shou, and Huang (2011) assumes that the leasing (wholesale) price and the service (retail) price are the same and it bases the assumption on the need to avoid arbitrage. We strongly argue against the above assumption and we therefore model the leasing price and the subscription price in separate stages.

4 Model

We assume that the operators compete *à la Bertrand*, that is, playing a one-shot simultaneous game where MO and FO strategies are p_1 and p_2, respectively.

Both the price p and the leased spectrum amount b are partially the result of a bargaining process between the MO and the FO.

Finally, each user will subscribe to the service providing the highest utility. The subscription period matches the time period of the bargain price p and the offered prices p_1 and p_2. Assuming that the number of users (n) is high enough, the individual subscription decision of each user will not affect the utility perceived by the rest, and a Wardrop equilibrium will result.

Thus, we model the strategic interaction between the two operators and the n users as a three-stage multi leader-follower game, which comprises three phases:

First phase: the bargain between the MO and the FO takes place, which may result in an outcome which gives a set of price p and amount b of the leased spectrum.

Second phase: the MO offers a subscription price p_1 and the FO offers p_2.

Third phase: each user chooses whether to subscribe to the service or not; if she chooses to subscribe, she simultaneously chooses which operator to subscribe to. The number of users that subscribe with operator i is denoted by n_i.

A standard way to analyse this sort of games is by means of backward induction.

4.1 Operators profit

The operators profits can be expressed as:

Macrocell operator

$$\Pi_1 = n_1 \cdot p_1 + p \cdot b - C_1 \tag{1}$$

Femtocell operator

$$\Pi_2 = n_2 \cdot p_2 - p \cdot b - C_2 \tag{2}$$

where C_i is the costs beared by operator i.

4.2 Subscription

In this phase, a pair of values b and p has been agreed and the prices p_1 and p_2 have been announced.

The utility that the users receive from each operator depends on two factors:

Quality of service Each operator exploits, during each subscription period, an amount of spectrum which is agreed at the end of the first phase: $W_2 = b$ for the FO and $W_1 = W - b$ for the MO. Different users experience different channel conditions to the macrocell base stations due to different locations, and thus achieve different data rates when using the same amount of bandwidth. We model this fact by a *macrocell spectrum efficiency* θ.

When using the femtocell service, since femtocell base stations are deployed indoors and are very close to the users' cell phones, we assume that all users using the femtocell service have equal good channel conditions and achieve the same maximum spectrum efficiency.

Bearing in mind the above issues, the product of the spectrum times the spectral efficiency for each operator, divided by the number of users which subscribe to it will give the transfer rate that is offered to each user (Niyato and Hossain 2008). We propose to use this transfer rate as the main quality factor which contributes to the user utility.

Price: the higher the subscription price, the lower the user utility.

Based on the above discussion, we propose a quasi-linear expression for the user utility. Specifically, for operator i's subscriber, it is

$$U_i = \log\left(1 + \frac{\theta_i W_i}{n_i}\right) - p_i. \tag{3}$$

where $\theta_2 = 1$, for the FO, and $\theta_1 < \theta_2$ for the MO.

Each user will subscribe to the operator providing the service with the higher utility. Assuming that the number of users is high enough, the individual subscription decision of each individual user will not affect the utility received by the rest. Then, the equilibrium notion is the Wardrop equilibrium, where each user makes her decision so that she is indifferent to choosing one alternative among all the available alternatives, and therefore no user has an incentive to change her decision.

Each user has three subscription alternatives:

1. to subscribe to the MO,
2. to subscribe to the FO or
3. not to subscribe to either.

We assume that the users who do not subscribe have a utility equal to 0. Applying the Wardrop equilibrium concept, we may state the following:

- if some users decide not to subscribe to either the MO or the FO, then $U_1 = U_2 = 0$, i.e., the users are indifferent between subscribing or not.

- alternatively, if every user subscribes to either the MO or the FO, then $U_i \geq 0$
  - if $U_1 > U_2$, then no user will subscribe to the FO, and all users will subscribe to the MO
  - if $U_1 = U_2$, then users will distribute between the MO and the FO
  - if $U_1 < U_2$, then no user will subscribe to the MO, and all users will subscribe to the FO

We address the first scenario, and show that actually it cannot occur in the equilibrium. Let us assume that $U_1 = U_2 = 0$ and some users do not subscribe to either operator. Then, from (3) it follows that at the user equilibrium, the number of subscribers n_i will be given by

$$n_i = \frac{\theta_i W_i}{e^{p_i} - 1} \,. \tag{4}$$

It can be observed that the number of subscribers each operator obtains solely depends on its subscription price and not on that of the competitor. This fact can be explained as follows. If one operator lowered the price p_i, it would attract users who would not have subscribed to the service otherwise, but it would not attract any subscriber from the competitor.

Anticipating the above user equilibrium, each operator will choose the value of p_i so that its profits are maximized. From (1)–(2) and (4) we have that

$$\frac{\partial \Pi_i}{\partial p_i} = \theta_i W_i \frac{(1-p_i)e^{p_i} - 1}{(e^{p_i} - 1)^2} < 0 \,, \qquad \text{if } p_i > 0 \,,$$

which indicates that by lowering the price, the operator gets more customers and increases the profits. Hence, the operator would prefer to lower the price as long as there are non-subscribing users, which eventually leads to the first scenario boiling down to the second scenario, where all users subscribe to the service.

The second scenario will be assumed in the next sections. As regards the three alternatives enumerated above, we will restrict in next section to the case where $U_1 = U_2$. The case $U_1 > U_2$ where no user subscribes to the FO cannot be an equilibrium, apart from the trivial case $b = 0$. From (3), and assuming finite prices, the utility U_2 that a user will potentially obtain from switching to the FO will be arbitrarily large. But this contradicts the fact that U_2 is bounded by U_1. Analogously, the case $U_2 > U_1$ where no user subscribes to the MO cannot be an equilibrium, apart from the trivial case $b = W$.

4.3 Competition

In this phase, a pair of values b and p has been agreed, and each operator chooses its strategy, which is the subscription price p_i, aiming to maximise its profits. Each operator does not know the strategy chosen by its competitor, but common knowledge of the strategies available to them and the corresponding profits is assumed.

Here it is assumed that we are under scenario 2 as defined above, that is, all users subscribe with one of the operators ($n_1 + n_2 = n$) and both operators have customers ($U_1 = U_2$). Let us denote by $\alpha = n_1/n$ the fraction of customers that subscribe to operator 1. We can then express the number of customers with each operator as a function of α

$$n_1 = \alpha n \tag{5}$$
$$n_2 = (1-\alpha)n\,. \tag{6}$$

To simplify notation we introduce

$$r_i = \frac{\theta_i W_i}{n}, \qquad i = 1, 2\,. \tag{7}$$

Using the equilibrium equation ($U_1 = U_2$)

$$\log\left(1 + \frac{r_1}{\alpha}\right) - p_1 = \log\left(1 + \frac{r_2}{(1-\alpha)}\right) - p_2 \tag{8}$$

and (5)–(6), n_1 and n_2 can be expressed as functions of p_1 and p_2, so that:

$$\Pi_1 = \Pi_1(p_1, p_2) \qquad\qquad \Pi_2 = \Pi_2(p_1, p_2). \tag{9}$$

The equilibrium strategies p_1^* and p_2^* will be given by the Nash equilibrium conditions:

$$\Pi_1(p_1^*, p_2^*) \geq \Pi_1(p_1, p_2^*), \quad \forall p_1, \tag{10}$$
$$\Pi_2(p_1^*, p_2^*) \geq \Pi_2(p_1^*, p_2), \quad \forall p_2. \tag{11}$$

Under the assumption that the partial derivatives of Π_1 and of Π_2 exist, the Nash equilibrium will be among the solutions of the equation system:

$$0 = \frac{\partial \Pi_1(p_1^*, p_2^*)}{\partial p_1} = n\left(\alpha + p_1^* \frac{\partial \alpha}{\partial p_1}\right), \tag{12}$$
$$0 = \frac{\partial \Pi_2(p_1^*, p_2^*)}{\partial p_2} = n\left((1-\alpha) - p_2^* \frac{\partial \alpha}{\partial p_2}\right), \tag{13}$$

which yields

$$p_1 = \alpha \left(-\frac{\partial \alpha}{\partial p_1}\right)^{-1} \tag{14}$$
$$p_2 = (1-\alpha)\left(\frac{\partial \alpha}{\partial p_2}\right)^{-1}. \tag{15}$$

Now, by taking derivatives with respect to p_1 and p_2 in (8) we obtain

$$\frac{\partial \alpha}{\partial p_2} = -\frac{\partial \alpha}{\partial p_1} = \left(\frac{1}{\alpha^2/r_1 + \alpha} + \frac{1}{(1-\alpha)^2/r_2 + 1 - \alpha}\right)^{-1}. \tag{16}$$

Combining (16) with (14)-(15) gives

$$\frac{p_1}{\alpha} = \frac{p_2}{(1-\alpha)} = \frac{1}{\alpha^2/r_1 + \alpha} + \frac{1}{(1-\alpha)^2/r_2 + 1 - \alpha}\,. \tag{17}$$

From here p_1 and p_1 can be expressed as functions of α, which substituted into (8) give

$$\log\left(1+\frac{r_1}{\alpha}\right)-\alpha\left(\frac{1}{\alpha^2/r_1+\alpha}+\frac{1}{(1-\alpha)^2/r_2+1-\alpha}\right)=$$
$$\log\left(1+\frac{r_2}{(1-\alpha)}\right)-(1-\alpha)\left(\frac{1}{\alpha^2/r_1+\alpha}+\frac{1}{(1-\alpha)^2/r_2+1-\alpha}\right). \quad (18)$$

Introducing

$$f(x,a,b)=\log\left(1+\frac{a}{x}\right)-\alpha\left(\frac{1}{x^2/a+x}+\frac{1}{(1-x)^2/b+1-x}\right) \quad (19)$$

we can rewrite (18) as

$$f(\alpha,r_1,r_2)=f(1-\alpha,r_2,r_1). \quad (20)$$

It is not difficult to check that as α varies in the interval $(0,1)$ $f(\alpha,r_1,r_2)$ monotonically decreases from $-\infty$ to ∞ and $f(1-\alpha,r_2,r_1)$ monotonically increases from $-\infty$ to ∞. Therefore, there exists a unique value $\alpha^*\in(0,1)$ which satisfies (18).

The analysis of equilibrium in the competition setting can be formulated more generally as a pair of optimization problems

$$\begin{cases}\max\limits_{\alpha,p_1} & f_1(\alpha,p_1)=\alpha p_1\\ \text{subject to} & g_1(\alpha,p_1)=U_1-U_2=0\\ & h_1(\alpha,p_1)=U_1\geq 0\end{cases} \quad \text{and} \quad \begin{cases}\max\limits_{\alpha,p_2} & f_2(\alpha,p_2)=(1-\alpha)p_2\\ \text{subject to} & g_2(\alpha,p_2)=U_1-U_2=0\\ & h_2(\alpha,p_2)=U_2\geq 0\end{cases}$$

Note that we have introduced α as an auxiliary optimization variable. In the second phase of the game the strategy of each operator consists of only its subscription price. The constraint $U_1=U_2$ defines α as a (decreasing) function of p_1 for a given value of p_2, and as a function of p_2 for a given value of p_1.

The Karush–Kuhn–Tucker (KKT) conditions for those two problems are

$$\nabla f_1+\lambda_1\nabla g_1+\mu_1\nabla h_1=0 \quad (21)$$
$$g_1=0 \quad (22)$$
$$h_1\geq 0 \quad (23)$$
$$\mu_1 h_1=0 \quad (24)$$
$$\mu_1\geq 0 \quad (25)$$

and

$$\nabla f_2+\lambda_2\nabla g_2+\mu_2\nabla h_2=0 \quad (26)$$
$$g_2=0 \quad (27)$$
$$h_2\geq 0 \quad (28)$$
$$\mu_2 h_2=0 \quad (29)$$
$$\mu_2\geq 0 \quad (30)$$

In this section we have so far implicitly assumed that $U_1=U_2>0$, so that the inequality constraints are not active and the optimization problems are the ones we already solved and whose solution is given by (18) and (14)-(15). That solution, however, yields $U_1 = U_2 < 0$ for some settings and in that cases the inequality constraint cannot bet let out. Now we examine those cases with active inequality constraints, i.e., $h_1 = 0$ or $h_2 = 0$.

If $h_1 = 0$, from (23) it follows that

$$p_1 = \log\left(1 + \frac{r_1}{\alpha}\right), \tag{31}$$

which plugged into (22) yields

$$p_2 = \log\left(1 + \frac{r_2}{1-\alpha}\right). \tag{32}$$

Finally, from (21) we obtain the following condition

$$f(\alpha, r_1, r_2) = -\frac{\mu_1}{(1-\alpha)^2/r_2 + 1 - \alpha} \leq 0, \tag{33}$$

where the function f is defined as in (19).

Likewise, if $h_2 = 0$ from (26)–(30) we obtain again Eqs. (31)-(32) and the condition

$$f(1-\alpha, r_2, r_1) = -\frac{\mu_2}{\alpha^2/r_1 + \alpha} \leq 0. \tag{34}$$

Let α_1 and α_2 be, respectively, the only solutions in $(0,1)$ of $f(\alpha, r_1, r_2) = 0$ and $f(1-\alpha, r_2, r_1) = 0$. As noted above, $f(\alpha, r_1, r_2)$ is monotonically decreasing from $-\infty$ to ∞ and $f(1-\alpha, r_2, r_1)$ monotonically increasing from $-\infty$ to ∞ so α_1 and α_2 are well defined. Moreover, condition (33) holds if, and only if, $\alpha \geq \alpha_1$; and (34) holds if, and only if, $\alpha \leq \alpha_2$.

Depending on the relative position of α_1 and α_2 we have one of the following three situations:

- If $\alpha_1 > \alpha_2$ it is not possible to meet conditions (33) and (34) simultaneously. Hence, there does not exist an equilibrium such that $U_1 = U_2 = 0$. Furthermore, the solution to (18) $\alpha^* \in (\alpha_2, \alpha_1)$ yields $U_1 = U_2 > 0$.
- If $\alpha_1 < \alpha_2$ any $\alpha \in (\alpha_1, \alpha_2)$ satisfies (33) and (34), which substituted into (31) and (32) yields a pair of prices that is an equilibrium point. Therefore, there exists an infinite and non-denumerable set of equilibrium points.
- If $\alpha_1 = \alpha_2$, then $\alpha^* = \alpha_1 = \alpha_2$ is both the only solution to (18) and the only value that satisfies (33) and (34) simultaneously.

4.4 Operators bargaining

As stated at the beginning of the section, the price p and the amount of spectrum b are subject to a bargaining between the MO and the FO, which is conducted before the subscription prices are advertised by the operators and the subscription decision is made by the users.

Following Mukherjee and Pennings (2006), we model the bargaining as a non-cooperative game where the incumbent operator—the MO—has full bargaining power and therefore offers a take-it-or-leave-it offer to the entrant operator—the FO. The offer consists of a pair of values p and b which the FO may accept or refuse. If the FO accepts the offer, then the competition and subscription phase do take place; else, the FO does not enter into the market, and the MO remains as the monopolist operator.

The bargaining is then proposed to be modeled as a dynamic game in a extensive form:

1. The MO operator offers a pair of values p and b
2. The FO operator may accept or refuse the offer.
   (a) If the FO accepts it, the operators will obtain profits which depends on the competition and the subscription phase, as modeled above and denoted as Π_1 and Π_2.
   (b) If the FO refuses it, the FO does not enter into the market —and therefore obtains zero profits, and the MO remains as the monopolist operator and obtains profits denoted as Π_m.

Following backward induction, depending on the values of (b, p) which characterise the MO offer, the bargaining outcome will be one or the other:

1. In order that the MO makes an offer, it must indeed prefer the competition outcome rather than the monopolist outcome, that is, $\Pi_1 \geq \Pi_m$.
2. If the offer made by the MO complies with

$$\Pi_2 \geq 0, \tag{35}$$

   the FO will accept the offer. Otherwise, $\Pi_2 < 0$, and the FO will refuse it.

To sum up, the FO operator will enter into the market and competition will be observed in the service provision to the users if the offer (b, p) made by the MO operator is such that:

$$\Pi_1 \geq \Pi_m \qquad \text{and} \qquad \Pi_2 \geq 0, \tag{36}$$

In other words, that both the MO and the FO are better off than it would be the case if the former remains as the monopolist and the latter refrains from entering into the market.

4.5 Monopoly profits

In the previous section, when deciding which offer to make to the FO, the MO compared profits in the competitive environment with profits in the monopolistic case. Thus, we need to compute Π_m.

In order to compute the profits Π_m, the expressions for the profits (1) and the user utility (3) will be computed taking $b = 0$ and substituting p_m for p_1:

- Profits are now given by

$$\Pi_m = n_1 \cdot p_m - C_1. \tag{37}$$

- User utility is given by

$$U_1 = \log\left(\theta_1 \frac{W}{n_1}\right) - p_m. \tag{38}$$

To compute Π_m, the problem should be stated as an optimal decision problem, such that the optimal price p_m^* should fulfill:

$$\Pi_m(p_m^*) \geq \Pi_m(p_m)\,, \qquad \forall p_m.$$

From the perspective of the MO the situation here is, in a sense, similar to the first scenario we ruled out in Section 4.2, where there would have been no competition between operators. There we concluded that the best interest for the operator was to get as many customers as possible by setting the price appropriately. Applying the same reasoning here, we obtain that the optimal price is

$$p_m^* = \log\left(1 + \frac{\theta_1 W}{n}\right)\,, \tag{39}$$

and then we obtain the value for the monopoly profits.

$$\Pi_m(p_m^*) = n \log\left(1 + \frac{\theta_1 W}{n}\right) - C_1\,. \tag{40}$$

We assume that the monopoly profits are positive. Note that $\Pi_m(p_m^*)$ is increasing with n and

$$\lim_{n\to\infty} \Pi_m(p_m^*) = \theta_1 W - C_1\,. \tag{41}$$

Consequently, $\theta_1 W > C_1$ is a necessary condition for the operator being able to make positive profits ($\Pi_m(p_m^*) > 0$). Moreover, even if that condition holds, a minimum number of users n is still required so that the operator makes some profits.

4.6 Bargaining outcome

From the conditions for the competitive equilibrium (see equation (36)) and using the expression for the monopoly profits in (40), we will proceed to define the feasibility region of values (b, p) which allows for the competitive equilibrium to result.

From the condition $\Pi_2 \geq 0$, we derive:

$$p \leq U(b) = \frac{n}{b}\left((1-\alpha)p_2 - C_2/n\right)\,, \tag{42}$$

and from $\Pi_1 \geq \Pi_m$:

$$p \geq L(b) = \frac{n}{b}\left(\log(1 + \frac{\theta_1 W}{n}) - \alpha p_1\right)\,. \tag{43}$$

Note that in the above expressions for $L(b)$ and $U(b)$ the values of α, p_1 and p_2 do also depend on b.

Therefore, for an amount of leased spectrum b competition could occur if and only if

$$L(b) \leq U(b), \tag{44}$$

which using (42) and (43) can be written as

$$\alpha p_1 + (1-\alpha)p_2 \geq \log(1 + \frac{\theta_1 W}{n}) - C_2/n. \tag{45}$$

Furthermore, a leased spectrum of b at a price p will lead to competition, i.e. the point (b, p) will be in the feasibility region, if and only if,

$$\max\big(0, L(b)\big) \leq p \leq U(b). \tag{46}$$

The final bargaining outcome will depend on the specific assumptions made over the bargaining process. If the incumbent has full bargaining power, as stated above in 4.4, then the value p will be such that the equality holds in (42), i.e.

$$p = U(b) \tag{47}$$

Given that Π_1 is monotonically increasing on the value p (see equation (1)), this would provide the incumbent with maximum profits.

By noting that

$$\Pi_1\Big|_{p=L(b)} = \Pi_m \quad \text{and} \tag{48}$$

$$\Pi_2\Big|_{p=U(b)} = 0\,, \tag{49}$$

we can rewrite (1) and (2) as

$$\Pi_1 = \Pi_m + \big(p - L(b)\big)b\,, \tag{50}$$

$$\Pi_2 = U(b) - p\,, \tag{51}$$

from what it follows that

$$\Pi_1 + \Pi_2 = \Pi_m + \Delta\,, \tag{52}$$

where $\Delta = \big(U(b) - L(b)\big)b \geq 0$ is the amount by which the total profit is incremented if the FO enters the market. Note that Δ does not depend on p. Note also that the problem of deciding how this extra profit is shared between the MO and the FO is equivalent to setting the price p at which the MO sells bandwidth to the FO.

For this problem multiple solution concepts can be borrowed from the cooperative game theory. For instance, the situation when the FO enters the market by leasing some bandwidth from the MO may be regarded as a coalition between the two. The total payoff of the coalition will be $\Pi_m + \Delta$. Therefore, by agreeing to share Δ both players will be better off than if each of them plays by itself, where the profit of the MO will be Π_m and that of the FO 0. Note that in this context the MO and the FO forming a coalition means that both agree on a price p but from that point on they compete in a non-cooperative fashion in the retailer game by setting their subscription fees —p_1 and p_2— independently from each other. In this context, the *Shapley value* provide a *fair* allocation of the payoff obtained by the coalition. In our case, the Shapley value allocation yields

$$\Pi_1 = \frac{1}{2}\Pi_m + \frac{1}{2}(\Pi_m + \Delta - 0) = \Pi_m + \frac{\Delta}{2}\,, \tag{53}$$

$$\Pi_2 = \frac{1}{2}0 + \frac{1}{2}(\Pi_m + \Delta - \Pi_m) = \frac{\Delta}{2}\,, \tag{54}$$

which correspond to

$$p = \frac{U(b) + L(b)}{2} . \tag{55}$$

Alternatively, the problem of agreeing a value for p can be casted into a two person bargaining problem in which the disagreement point is $(\Pi_m, 0)$ and the players' strategies are their offers about p. In this setting, both the *Nash bargaining solution* and the *Kalai-Smorodinsky bargaining solution* yield the same results as the provided by the Shapley value (see (53) and (54)).

Note, however, that the value b is not determined by the bargaining, but only constrained by (44) .

5 Solution

Based on the model developed in the previous section, we propose the following solving procedure:

1. The following parameters are fixed: n, W, C_1, C_2, θ_1, and θ_2.

2. We propose a value b, such that $0 < b \leq W$, and assume that (45) holds.

3. We compute the solutions to (33) and to (34). Only if both solutions are equal, there exists a unique α^* solving (18), and we proceed to compute the equilibrium values.

4. We obtain the equilibrium strategies for each operator $\hat{p_1}$ and $\hat{p_2}$, using (17).

5. Condition (45) is tested.

   (a) If it is false, the outcome is that MO remains as monopolist and FO does not enter:

      i. For the MO operator,

         A. $\alpha^* = 1$

         B. $p_1^* = p_m^*$, as given by (39)

         C. $\Pi_1 = \Pi_m$, as given by (40)

      ii. For the FO operator, $p_2^* = 0$, and $\Pi_2 = 0$

   (b) If it is true, we set $p_i^* = \hat{p}_i$ and compute U_i. Furthermore, a value of p is chosen according to the assumptions made in section 4.6. Profits for MO and for FO, Π_1 and Π_2, are computed using expressions (1) and (2), respectively.

In order to evaluate the different competitive equilibria, we propose to use the following indicators:

1. Operators profits Π_1 and Π_2,

2. User utilities, $U_1 = U_2$[3],

3. User welfare, UW, computed as the aggregated user utility over the total number of subscribers:

$$UW = n \cdot U_1 = n \cdot U_2 \tag{56}$$

[3]This equality holds for the competitive equilibrium

Parameter	Value
n	10000 users
W	95 kHz
C_1	20 m.u.
C_2	10 m.u.
θ_1	0.5 bit/s/Hz
θ_2	1 bit/s/Hz

Tabla 1: Parameters

4. Social welfare, SW, computed as the sum of the user welfare and the producer welfare:
$$SW = UW + \Pi_1 + \Pi_2 \tag{57}$$

6 Results

In this section, the numerical results obtained when following the solving procedure outlined in the previous section are presented. First, the parameter values are listed. Second, the equilibrium for each parameter configuration is represented in graphs. And third, the results are analysed.

6.1 Parameters

The values for the parameters, if not stated otherwise, are the ones shown in table 1.

6.2 Experiment no. 1

Objective : To characterise the feasibility region for (b, p).

Constant parameters : n, W, C_1, C_2, θ_1, and θ_2

Variable parameters : none

Control variable : b

Observed variable :

- $U(b)$, given by (42).
- $L(b)$, given by (43).

MATLAB script : `TELPOL23e\scriptFigs-separades.m`

Graphs : See Fig 2.

Conclusions : We see that:

- For values of b greater than a threshold value b^f_{min}, $L(b) < U(b)$ holds and, therefore, corresponding values for p can be found such that competition results in an equilibrium.

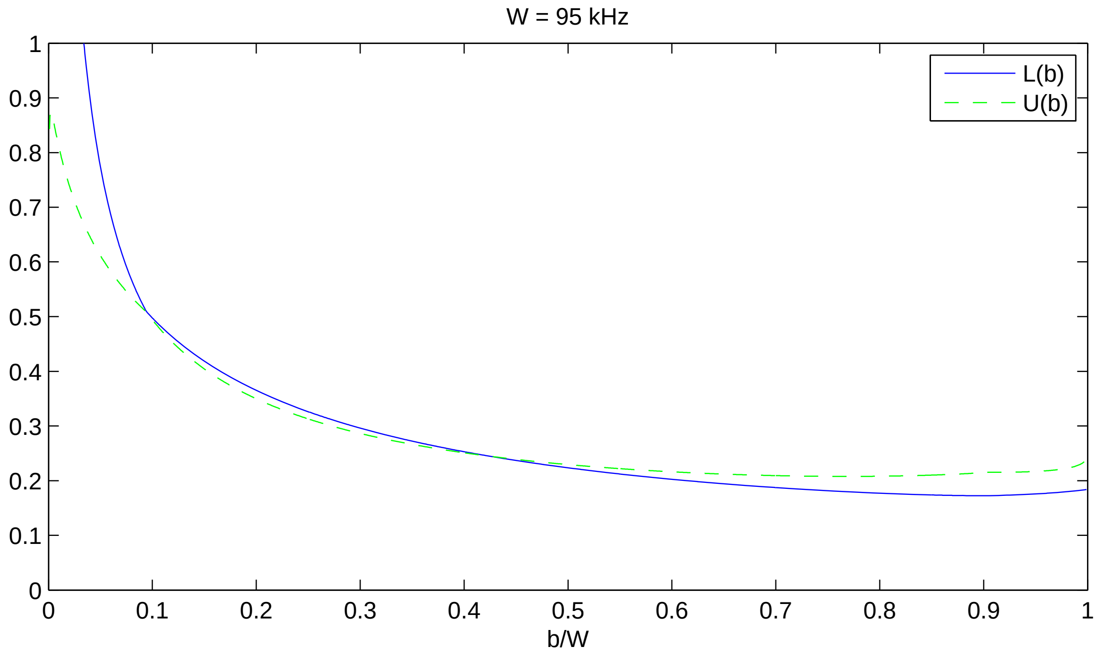

Figure 2: Feasibility values for (b, p)

6.3 Experiment no. 3

Objective : To check the effect of varying the leased amount b of spectrum. We assumed that p is agreed so that the Shapley values result for the profits.

Constant parameters : n, W, C_1, C_2, θ_1, and θ_2.

Variable parameters : none

Control variable : b

Observed variables :

- α
- U_1 and U_2
- Π_1 and Π_2

MATLAB script :`TELPOL23e\scriptFigs-separades.m`

Graphs : See Fig. 3, Fig. 4, and Fig. 5.

For the values b such that multiple equilibria result, i.e., $b \notin [b^u_{min}, b^u_{max}]$, no value is represented.

For the values of b such that (b, p) does not yield competitive equilibrium, i.e., $b < b^f_{min}$, results for the MO correspond to the monopoly scenario.

Conclusions We see that:

- With respect to the number of subscribers:

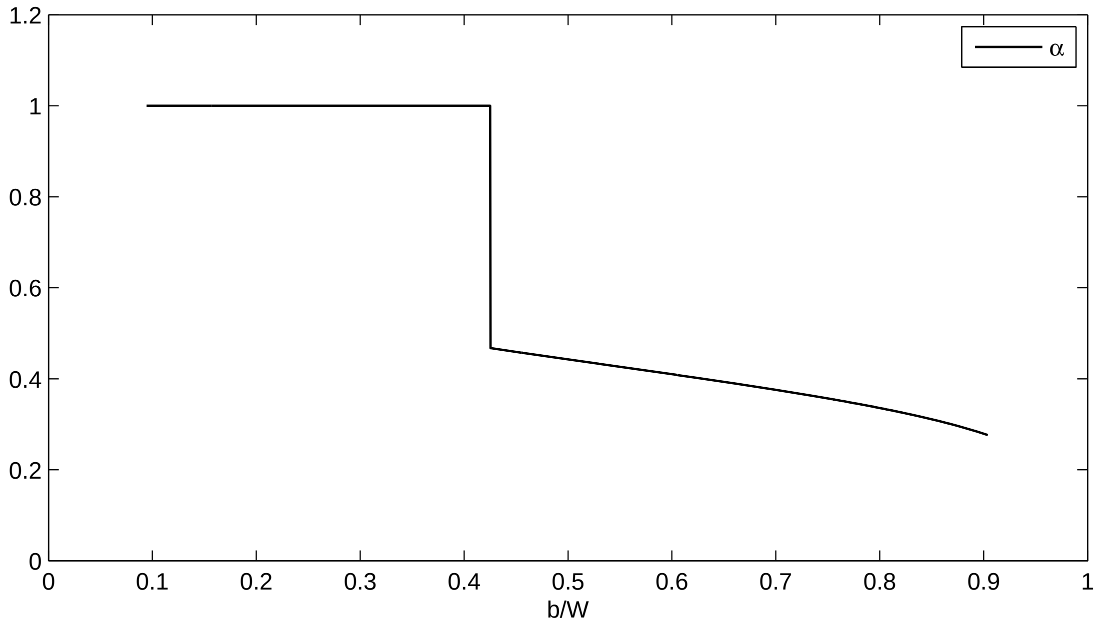

Figure 3: Fraction of MO users over the total number of subscribers (α)

  - When the MO remains as the monopolistic operator, $\alpha = 1$ holds. When the FO operator enters the market, i.e. $b \geq b^u_{min}$ and $b \geq b^f_{min}$, the MO operator looses market share as b increases—while $b < b^u_{max}$.

- With respect to utilities U_1 and U_2:
  - As stated in section 4.3, the range of values where a unique equilibrium results corresponds to those values with positive values of $U_1 = U_2$.
  - The entry of FO is beneficial for the users, as the utility increases as b increases up to a value $b_{UW} = 0.5W$. From this value, the utility decreases down to zero. We may conclude from this behavior that a symmetric equilibrium, where the FO and the MO have the same amount of resources (the spectrum $W/2$), precludes the MO and the FO to exercise any kind of market power. On the contrary, the level of competition is maximum and the users derive the maximum utility from the service.

- With respect to profits Π_1 and Π_2:
  - Both operators increase their respective profits when the FO operator enters the market. Furthermore, they keep increasing as the FO gets more resources (the spectrum b) for providing service to its users. The FO's higher efficiency and the increasing payment to the MO explains this behavior.

- With respect to the social welfare SW:
  - A maximum is reached for a value b_{SW}, which is greater than b_{UW}. Bearing in mind that the social welfare adds up the users utility and the operators profits, this maximum is a trade off between the maximum utility reached at b_{UW} and the increasing profits.

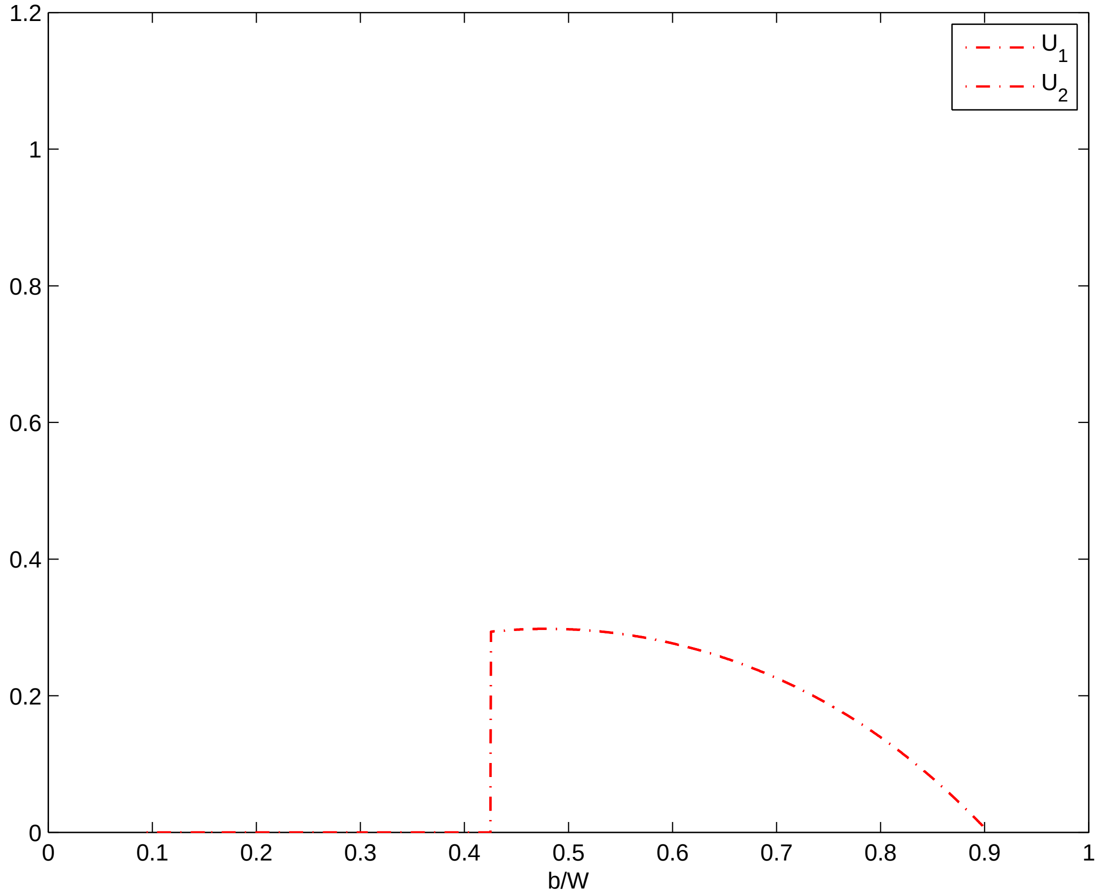

Figure 4: Utility for subscribers for MO and FO

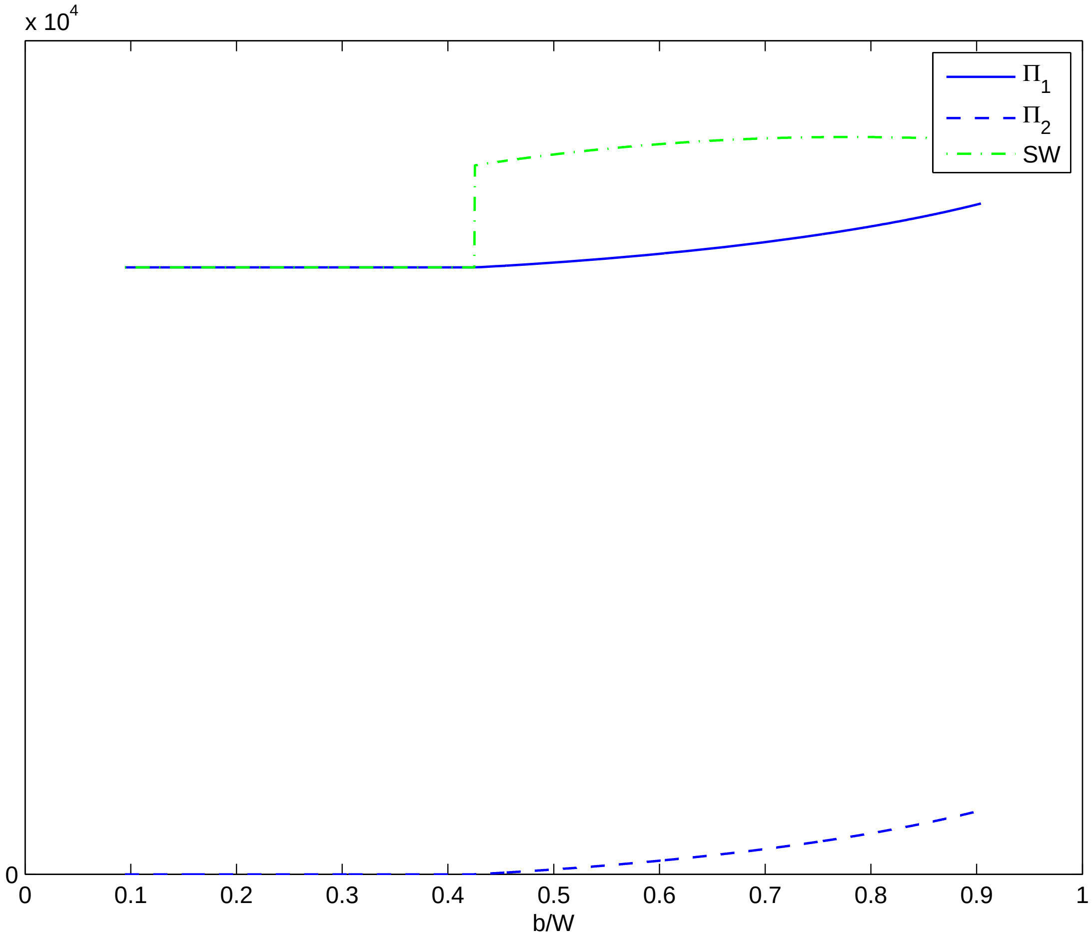

Figure 5: Profits for MO and FO

The above conclusions are valid regardless the criteria used for fixing p. In this experiment, the value p that is agreed provides Shapley values for the profits. For any other value of p, specifically for $p = U(b)$ —i.e. the incumbent has full bargaining power—the profits Π_1 and Π_2 would obviously be different. Nevertheless, the producer welfare and the equilibrium prices remain the same as those just represented and discussed, and consequently the user utility, the user welfare and the social welfare.

6.4 Experiment no. 4

Objective : To evaluate the optimum values b of leased spectrum from the point of view of the welfare.

Constant parameters : n, C_1, C_2, θ_1 and θ_2

Variable parameters : none

Control variable : W

Observed variables :

- Maximum value of b/W in the feasibility curve which results in a unique competitive equilibrium[4](b_{max}/W).
- Minimum value of b/W in the feasibility curve which results in a unique competitive equilibrium (b_{min}/W).
- Value of b/W between b_{min}/W and b_{max}/W such that user welfare is maximised (b_{UW}/W).
- Value of b/W between b_{min}/W and b_{max}/W such that social welfare is maximised (b_{SW}/W)

MATLAB script :`TELPOL23e\script_caracteritzacio.m`

Graphs : See Fig. 6

Conclusions :

We see that:

- The value b_{max}/W tends to the value 1, which is the case where the whole spectrum W is leased to the FO and the profits are maximised. The constraint is given by the uniqueness of the equilibrium.
- As regards b_{min}/W, there is a lower range of values of W where the uniqueness criteria constraints the possibility of a competitive equilibrium, $b_{min} = b^u_{min} > b^f_{min}$, and a higher range of values of W where the bargaining constraints the possibility, $b_{min} = b^f_{min} > b^u_{min}$.
- As explained in section 6.3, b_{UW} is lower than b_{SW}, and they almost keep constant until b_{min} increases and precludes any interior maximum b_{UW} and b_{SW} to occur.

[4]The feasibility region, when p is computed from (55), becomes a curve

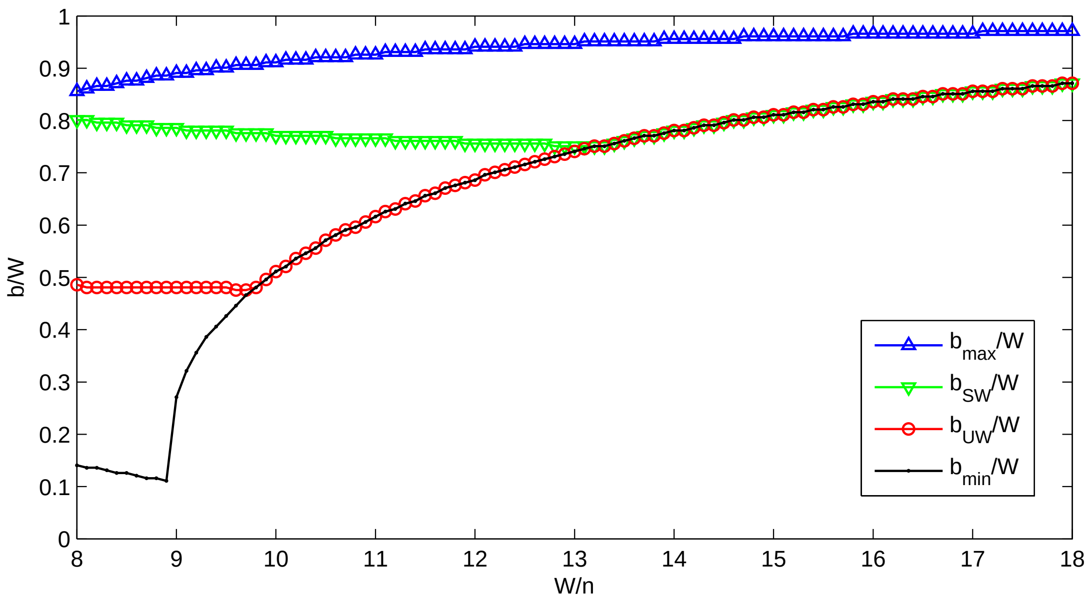

Figure 6: Leased spectrum values which yield maximum user welfare and maximum social welfare

The above results mean that the degenerate case $b/W = 1$, which would be the optimum from the point of view of the producer welfare, is not always the optimum from the point of view of either user welfare or social welfare. We would argue then that a regulatory authority would have strong arguments — i.e., welfare enhancement— to intervene by fixing a maximum value $b/W < 1$ of leased spectrum. And these arguments are independent on the procedure that implements the bargaining on p.

7 Conclusions

Bearing in mind the analysis of the results conducted in the previous section, we can conclude that:

1. Every actor, that is, users and the two operators, are better off when the FO operator enters the market —which requires to be within the feasibility and uniqueness region.

2. The regulator intervention is deemed necessary in order to restrain the incumbent operator from leasing the whole amount of the spectrum to the entrant operator, which will harm the users.

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