Transformada de Laplace

Ejercicios 6 – 13 (página 142 del libro). **Calcula la Transformada de Laplace de las siguientes funciones**

```
Clear[f1, f2, f3, f4, f5, f6, f7, f8, f9, f10, f11, f12, f13]
```

```
f1[t_] = 1 / (4 t^(3 / 5)) + 1 / (2 t^(1 / 10)) - t^(3 / 20)
f2[t_] = t Cos[t]
f3[t_] = t Sin[t]
f4[t_] = E^(3 t) Cos[4 t]
f5[t_] = Integrate[E^(3 x), {x, 0, t}]
f6[t_] = UnitStep[t] - UnitStep[t - 5]
f7[t_] = UnitStep[t - 1] - UnitStep[t - 2]
f8[t_] = UnitStep[t - 2 Pi / 3] Sin[t - 2 Pi / 3]
f9[t_] = (UnitStep[t - 1] - UnitStep[t - 3]) t^2
f10[t_] = 3 (UnitStep[t - Pi] - UnitStep[t]) + UnitStep[t - 2 Pi] Sin[t]
f11[t_] = E^(3 t) Cosh[4 t]
f12[t_] = t E^(3 t) Cosh[4 t]
f13[t_] = (t + 2)^2 E^t
```

$\frac{1}{4\,t^{3/5}} + \frac{1}{2\,t^{1/10}} - t^{3/20}$

t Cos[t]

t Sin[t]

e^{3t} Cos[4 t]

$\frac{1}{3}\left(-1 + e^{3t}\right)$

-UnitStep[-5 + t] + UnitStep[t]

-UnitStep[-2 + t] + UnitStep[-1 + t]

-Sin[$\frac{2\pi}{3}$ - t] UnitStep[-$\frac{2\pi}{3}$ + t]

t^2 (-UnitStep[-3 + t] + UnitStep[-1 + t])

Sin[t] UnitStep[-2 π + t] + 3 (-UnitStep[t] + UnitStep[-π + t])

e^{3t} Cosh[4 t]

e^{3t} t Cosh[4 t]

$e^{t}\,(2 + t)^2$


```
N[LaplaceTransform[f1[t], t, s]]
LaplaceTransform[f2[t], t, s]
LaplaceTransform[f3[t], t, s]
LaplaceTransform[f4[t], t, s]
LaplaceTransform[f5[t], t, s]
LaplaceTransform[f6[t], t, s]
LaplaceTransform[f7[t], t, s]
LaplaceTransform[f8[t], t, s]
LaplaceTransform[f9[t], t, s]
LaplaceTransform[f10[t], t, s]
LaplaceTransform[f11[t], t, s]
LaplaceTransform[f12[t], t, s]
LaplaceTransform[f13[t], t, s]
```

$-\frac{0.933041}{s^{23/20}} + \frac{0.534314}{s^{9/10}} + \frac{0.55454}{s^{2/5}}$

$\frac{-1 + s^2}{(1 + s^2)^2}$

$\frac{2\,s}{(1 + s^2)^2}$

$\frac{-3 + s}{16 + (-3 + s)^2}$

$\frac{1}{3}\left(\frac{1}{-3 + s} - \frac{1}{s}\right)$

$\frac{1}{s} - \frac{e^{-5s}}{s}$

$-\frac{e^{-2s}}{s} + \frac{e^{-s}}{s}$

$\frac{e^{-\frac{2\pi s}{3}}}{1 + s^2}$

$-\frac{e^{-3s}\,(2 + 6\,s + 9\,s^2)}{s^3} + \frac{e^{-s}\,(2 + s\,(2 + s))}{s^3}$

$-\frac{3}{s} + \frac{3\,e^{-\pi s}}{s} + \frac{e^{-2\pi s}}{1 + s^2}$

$\frac{-3 + s}{-7 - 6\,s + s^2}$

$\frac{1}{2}\left(\frac{1}{(-7 + s)^2} + \frac{1}{(1 + s)^2}\right)$

$\frac{2}{(-1 + s)^3} + \frac{4}{(-1 + s)^2} + \frac{4}{-1 + s}$

Ejercicio 18

```
Clear[f]
f[t_] = (Cos[a t] - Cos[b t]) / t
```

$\frac{\text{Cos}[a\,t] - \text{Cos}[b\,t]}{t}$

```
Simplify[LaplaceTransform[f[t], t, s]]
```

$$\frac{1}{2}\left(\text{Log}[a^2] - \text{Log}[b^2] - \text{Log}[a^2 + s^2] + \text{Log}[b^2 + s^2]\right)$$

Ejercicio 2. Calcula la Transformada inversa de Laplace de las siguientes funciones

Ejercicio 40 (página 145)

```
F1[s_] = 12 / (s^2 + 8);
F2[s_] = (2 s + 1) / (s^2 - 2 s + 2);
F3[s_] = (s + 5) / (s^3 - 11 s^2 + 31 s - 21);
F4[s_] = 1 / (3 s - 4) ^3;
F5[s_] = 1 / (s^2 + 1) ^2;
F6[s_] = (s^2 - 9) / (s^2 + 9) ^2;
F7[s_] = 1 / (s^2 (s^2 + 8));
F8[s_] = E^ (-2 s) / (s^2 - 4);
F9[s_] = (s^2 + 1) / (s - 1) ^3;
F10[s_] = s / (s + 1) ^4;
F11[s_] = (3 s^2 + 8 s - 1) / ((s - 3) (s + 2) ^2);
F12[s_] = s / ((s + 1) (s - 2) ^5);
F13[s_] = (5 s + 3) / (s^2 + 2) ^4;
F14[s_] = 54 / (s^4 - 81);
F15[s_] = 54 / (s^4 + 81);
```

```
Simplify[InverseLaplaceTransform[F1[s], s, t]]
Simplify[InverseLaplaceTransform[F2[s], s, t] // ComplexExpand]
Simplify[InverseLaplaceTransform[F3[s], s, t]]
Simplify[InverseLaplaceTransform[F4[s], s, t]]
Simplify[InverseLaplaceTransform[F5[s], s, t]]
```

$$3\sqrt{2}\,\text{Sin}\left[2\sqrt{2}\,t\right]$$

$$e^{t}\,(2\,\text{Cos}[t] + 3\,\text{Sin}[t])$$

$$\frac{1}{2}\,e^{t}\,(1 - 2\,e^{2t} + e^{6t})$$

$$\frac{1}{54}\,e^{4t/3}\,t^2$$

$$\frac{1}{2}\,(-t\,\text{Cos}[t] + \text{Sin}[t])$$


```
Simplify[InverseLaplaceTransform[F6[s], s, t]]
Simplify[InverseLaplaceTransform[F7[s], s, t]]
Simplify[InverseLaplaceTransform[F8[s], s, t]]
Simplify[InverseLaplaceTransform[F9[s], s, t]]
Simplify[InverseLaplaceTransform[F10[s], s, t]]
Simplify[InverseLaplaceTransform[F11[s], s, t]]
Simplify[InverseLaplaceTransform[F12[s], s, t]]
Simplify[InverseLaplaceTransform[F13[s], s, t]]
Simplify[InverseLaplaceTransform[F14[s], s, t]]
Simplify[InverseLaplaceTransform[F15[s], s, t] // ComplexExpand]
```

$$t\,\text{Cos}[3t]$$

$$\frac{1}{32}\left(4t - \sqrt{2}\,\text{Sin}\left[2\sqrt{2}\,t\right]\right)$$

$$\frac{1}{4}\,e^{4-2t}\,(-1 + e^{-8+4t})\,\text{UnitStep}[-2 + t]$$

$$e^{t}\,(1 + t)^2$$

$$-\frac{1}{6}\,e^{-t}\,(-3 + t)\,t^2$$

$$e^{-2t}\,(1 + 2\,e^{5t} + t)$$

$$\frac{1}{972}\,e^{-t}\,(4 + e^{3t}\,(-4 + 12t - 18t^2 + 18t^3 + 27t^4))$$

$$\frac{1}{768}\left(6t\,(-15 - 10t + 2t^2)\,\text{Cos}\left[\sqrt{2}\,t\right] + \sqrt{2}\,(45 + 30t - 36t^2 - 20t^3)\,\text{Sin}\left[\sqrt{2}\,t\right]\right)$$

$$\frac{1}{2}\,(-e^{-3t} + e^{3t} - 2\,\text{Sin}[3t])$$

$$\frac{e^{-\frac{3t}{\sqrt{2}}}\left(-\left(-1 + e^{3\sqrt{2}\,t}\right)\text{Cos}\left[\frac{3t}{\sqrt{2}}\right] + \left(1 + e^{3\sqrt{2}\,t}\right)\text{Sin}\left[\frac{3t}{\sqrt{2}}\right]\right)}{\sqrt{2}}$$

Ejercicio 42

```
g1[s_] = (s E^ (-s Pi / 2)) / (s^2 + 9);
g2[s_] = (1 - E^ (-2 s)) / s^2;
g3[s_] = E^ (-s Pi / 2) / (s^2 - 1);
g4[s_] = E^ (-4 s) / s^4;
g5[s_] = (1 - E^ (-s)) ^2 / s^3;
g6[s_] = E^ (-s Pi) / (s^2 + 9);
```

```
Simplify[InverseLaplaceTransform[g1[s], s, t]]
Simplify[InverseLaplaceTransform[g2[s], s, t]]
Simplify[InverseLaplaceTransform[g3[s], s, t]]
Simplify[InverseLaplaceTransform[g4[s], s, t]]
Simplify[InverseLaplaceTransform[g5[s], s, t]]
Simplify[InverseLaplaceTransform[g6[s], s, t]]
```

$$-\text{Sin}[3\,t]\ \text{UnitStep}\left[-\frac{\pi}{2}+t\right]$$

$$\begin{cases} 2 & t \geq 2 \\ t & \text{True} \end{cases}$$

$$\frac{1}{2}\,e^{-\frac{\pi}{2}-t}\,(-e^{\pi}+e^{2\,t})\ \text{UnitStep}\left[-\frac{\pi}{2}+t\right]$$

$$\frac{1}{6}\,(-4+t)^3\ \text{UnitStep}[-4+t]$$

$$\frac{1}{2}\,(t^2+(-2+t)^2\ \text{UnitStep}[-2+t]-2\,(-1+t)^2\ \text{UnitStep}[-1+t])$$

$$-\frac{1}{3}\ \text{Sin}[3\,t]\ \text{UnitStep}[-\pi+t]$$

Ejercicio 48

```
res = InverseLaplaceTransform[
  Solve[LaplaceTransform[x''[t] + 9 x[t] == 3 t, t, s] /. {x[0] → 0, x'[0] → 0},
   LaplaceTransform[x[t], t, s]], s, t]
```

$$\left\{\left\{x[t]\to\frac{1}{9}\,(3\,t-\text{Sin}[3\,t])\right\}\right\}$$

```
res = InverseLaplaceTransform[
  Solve[LaplaceTransform[x''[t] + 9 x[t] == 0, t, s] /. {x[0] → 1, x'[0] → 2},
   LaplaceTransform[x[t], t, s]], s, t]
```

$$\left\{\left\{x[t]\to\text{Cos}[3\,t]+\frac{2}{3}\,\text{Sin}[3\,t]\right\}\right\}$$

Ejercicio 53

a)

```
InverseLaplaceTransform[Solve[
  LaplaceTransform[x''[t] + x[t] == (UnitStep[t - 1] - UnitStep[t - 2]) (2 - t), t, s] /.
   {x[0] → 0, x'[0] → 0}, LaplaceTransform[x[t], t, s]], s, t]
```

$$\{\{x[t]\to (-2+t+\text{Sin}[2-t])\ \text{UnitStep}[-2+t]-(-2+t+\text{Cos}[1-t]+\text{Sin}[1-t])\ \text{UnitStep}[-1+t]\}\}$$

b)

```
InverseLaplaceTransform[Solve[
  LaplaceTransform[x''[t] + 4 x'[t] + 4 x[t] == t - UnitStep[t - 1] (t + E^(-2 (t - 1))),
    t, s] /. {x[0] → 1, x'[0] → 0}, LaplaceTransform[x[t], t, s]], s, t]
```

$$\left\{\left\{x[t]\to\frac{1}{4}\,e^{-2\,t}\,(5+e^{2\,t}\,(-1+t)+9\,t-(-1+t)\,(e^{2\,t}+e^2\,(-3+2\,t))\ \text{UnitStep}[-1+t])\right\}\right\}$$

c)


```
InverseLaplaceTransform[
 Solve[LaplaceTransform[x'[t] + x[t] == DiracDelta[t - 1], t, s] /. {x[0] → 1},
  LaplaceTransform[x[t], t, s]], s, t]
```

$$\{\{x[t]\to e^{-t}\,(1+e\ \text{UnitStep}[-1+t])\}\}$$

d)

```
InverseLaplaceTransform[
 Solve[LaplaceTransform[x''[t] + x[t] == DiracDelta[t - Pi] Cos[t], t, s] /.
   {x[0] → 0, x'[0] → 1}, LaplaceTransform[x[t], t, s]], s, t]
```

$$\{\{x[t]\to\text{Sin}[t]\,(1+\text{UnitStep}[-\pi+t])\}\}$$

e)

```
InverseLaplaceTransform[
 Solve[LaplaceTransform[x''[t] + 2 x'[t] + x[t] == DiracDelta[t - 1] + UnitStep[t - 2 Pi],
    t, s] /. {x[0] → 0, x'[0] → 1}, LaplaceTransform[x[t], t, s]], s, t]
```

$$\{\{x[t]\to e^{-t}\,(t+e\,(-1+t)\ \text{UnitStep}[-1+t]+(e^t+e^{2\,\pi}\,(-1+2\,\pi-t))\ \text{UnitStep}[-2\,\pi+t])\}\}$$

Ejercicio 56

```
InverseLaplaceTransform[Solve[LaplaceTransform[{x'[t] + x[t] + 2 y'[t] + 3 y[t] == 0,
     x'[t] - 4 x[t] + 3 y'[t] - 8 y[t] == Sin[t]}, t, s] /. {x[0] → 2, y[0] → -1},
  {LaplaceTransform[x[t], t, s], LaplaceTransform[y[t], t, s]}], s, t]
```

$$\left\{\left\{x[t]\to\frac{1}{3}\,(-2\,\text{Cos}[t]+8\,\text{Cos}[2\,t]-3\,\text{Sin}[t]+6\,\text{Sin}[2\,t]),\ y[t]\to\frac{1}{3}\,(-4\,\text{Cos}[2\,t]+\text{Sin}[t]-\text{Cos}[t]\,(-1+4\,\text{Sin}[t]))\right\}\right\}$$

Ejercicio 57

a)

```
InverseLaplaceTransform[
 Solve[LaplaceTransform[{x'[t] == -x[t] + y[t] + 1, y'[t] ==  x[t] - y[t] + 1}, t, s] /.
   {x[0] → 0, y[0] → 0},
  {LaplaceTransform[x[t], t, s], LaplaceTransform[y[t], t, s]}], s, t]
```

$$\{\{x[t]\to t,\ y[t]\to t\}\}$$

b)

```
InverseLaplaceTransform[
 Solve[LaplaceTransform[{x'[t] == y[t] + E^(-t), y'[t] == 4 x[t] + Sin[t]}, t, s] /.
   {x[0] → 0, y[0] → 0},
  {LaplaceTransform[x[t], t, s], LaplaceTransform[y[t], t, s]}], s, t]
```

$$\left\{\left\{x[t]\to\frac{1}{60}\,(-e^{-2\,t}\,(33-20\,e^t-13\,e^{4\,t})-12\,\text{Sin}[t]),\ y[t]\to\frac{1}{30}\,(-e^{-2\,t}\,(-33+40\,e^t-13\,e^{4\,t})-6\,\text{Cos}[t])\right\}\right\}$$

Ejercicio 59

```
DSolve[{t*y''[t] - t*y'[t] - y[t] == 0, y[0] == 0, y'[0] == 3}, y[t], t]
```

$$\{\{y[t]\to 3\,e^t\,t\}\}$$