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Automatic design of concrete vaults using iterated local search and extreme value estimation

Abstract

This paper describes an iterated local search algorithm based on a Gray code global best-descent (ILS-GB) for the automatic design and cost minimization of reinforced concrete vaults for road construction. The study involves a vault which measures 12.40 m in horizontal free span, 3.00 m in vertical height of the lateral walls and 1.00 m in earth cover. This problem includes 49 discrete design variables as well as penalty functions for unfeasible solutions. An objective methodology based on the extreme value theory is used to determine the number of experimental tests required to provide a solution with user-defined accuracy as compared to a global optimum solution. Results indicate that the local optima found by ILS-GB fits a three-parameter Weibull distribution so the estimated location parameter γ can be used as an estimation of the global minimum cost solution. The minimum value obtained by ILS-GB differed just 0.81% compared to the theoretical minimum value so that, from the structural engineering perspective, the divergence was small enough to be accepted. Finally, the optimization method indicates savings of about 7% compared to a traditional design.

Keywords

Concrete structures, extreme value theory, heuristic optimization, structural design, road vaults, Weibull distribution

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1 INTRODUCTION

Reinforced concrete (RC) vaults are essential in the construction of underpasses for roads, railways and waterworks. They are recommended when the length of the structure exceeds several hundred meters and the earth cover exceeds several meters. Typical free spans vary from a minimum of 4.00 m for small hydraulic sections to a maximum of 13.00 m for road and railway tunnels. The construction is usually done in segments of about 12.00 m in length. Characteristic features are the foundation slab, lateral walls and top semicircular vault. The current design of these structures is highly dependent upon the experience of the project designer. However, structural optimization methods provide an objective alternative to traditional design.

Applying optimization methods to the design of RC structures is deemed both appropriate and feasible since the element design is made more efficient. Generally speaking, optimization methods used in structural design are classified as mathematical programming and heuristic search methods. Sarma and Adeli [28] provided an extensive review of studies on non-heuristic structural concrete optimization, but now it is possible to use heuristic search methods to provide good solutions with reasonable computation times. These heuristic methods include many probabilistic-based search algorithms which were inspired by natural phenomena (natural evolution, physical processes or social interaction simulation among members of a specific species in search of food): genetic algorithms [15], simulated annealing [17], and ant colonies optimization [8], among others. In this context, there is an increasing effort on applying optimization procedures to the problem of sizing, shape or topology optimization structural problems [10,29]. However, after reviewing the optimization methods used in structural design, Cohn and Dinovitzer [6] emphasized the gap between theoretical and realistic applications, confirming that most research focused on steel structures, whereas only a small fraction dealt with RC structures.

The earliest studies into the optimization of structural concrete for beams date back to the late 1990s [1,5]. Many later studies used evolutionary programming, and in particular, genetic algorithms. Kicinger et al. [16] highlighted developments in evolutionary programming and structural design while the present authors' research group has recently reported on non-evolutionary techniques for optimization of retaining walls [30], bridge frames [26], building frames [23-25], prestressed concrete precast pedestrian bridges [19], and bridge piers [20,21].

Following this line of work, Carbonell et al. [3] developed a model for the optimum design of RC road vaults involving three types of neighborhood-based algorithms: a multi-start global best-descent local search, a meta-simulated annealing and a meta-threshold acceptance. These algorithms provide different results in each run due to the large number of random decisions and they all need a previous calibration. In contrast, this paper proposes an algorithm that does not require prior calibration, and also aims to determine the number of times the algorithm should be run to achieve sufficient accuracy. This method involves identifying an objective stopping criterion for a multi-start algorithm to reconcile the quality of the solution and the computation time required. If one accepts that the local optimum found by a stochastic search algorithm can be deemed as an extreme solution of a simple random sample consisting of solutions visited, then the Extreme Value Theory (EVT) can be applied to estimate the global optimum solution. The application of EVT to heuristic methods was described by McRoberts [22], Golden and Alt [14], and more recently Paya et al. [25].

To continue in this line of research, this study focuses on the economic optimization of road vault underpasses. To the best of our knowledge, the iterated local search (ILS) scheme has not yet been applied to optimize RC structures. In addition, this study applies a method to determine the minimum number of computer runs that an ILS algorithm must perform to ensure that the best result obtained does not differ more than a predetermined threshold with respect to the global optimum estimation using the EVT. The methodology consisted of developing a computer evaluation module in which cross-section dimensions, materials and steel reinforcement were used as discrete variables. The module computed the cost for a solution and checked all the relevant limit states. The cost objective function was then calculated. An ILS algorithm based on a global best-descent strategy (abbreviated herein as ITS-GB) was then used to search the soltuion space to identify a set of solutions with optimized values for the designer. The rest of this paper is organized as follows. In Section 2, we define the optimization problem while in Section 3, we explain the heuristic method developed. In Section 4, we describe the resulting computational experience with this algorithm, and in Section 5, we present the main conclusions and directions for future investigation.

2 THE OPTIMUM DESIGN PROBLEM

The structural design problem established for this study aims to minimize the cost of a RC vault, represented by the objective function F of Eq. (1), so that it meets the constraints contained in Eq. (2).

$$F(x_1, x_2, ..., x_n) = \sum_{i=1,r} p_i \cdot m_i \quad x_1, x_2, ..., x_n \tag{1}$$

$$g_j(x_1, x_2, ...x_n) \leq 0 \tag{2}$$

$$x_i \in \quad d_{i1}, d_{i2}, ..., d_{iq_i} \tag{3}$$

Note that x_1, x_2,..., x_n are the design variables whose combination is to be optimized. Each design variable can take on the discrete values listed in Eq. (3). The objective function F defined in Eq. (1) is the cost of the vault per linear meter (€/m), where p_i are the basic prices (Table 1); m_i are the measurements of the construction units (concrete, steel, formwork, etc.), and r is the total number of construction units. The constraints g_j in Eq. (2) are all the service limit states (SLSs) and ultimate limit states (ULSs) with which the structure must comply, as well as the geometrical and constructability constraints of the problem. The design variables and structural constraints considered in this study are described in full in a previously cited publication by Carbonell et al. [3]. They will only be summarized here for the readers convenience.

This study transforms constrained problems into unconstrained ones using the penalty function given by Eq. (4):

$$F^+ = F + \sum \Phi_j P_j \tag{4}$$

where F^+ represents the penalized cost; F is the cost; Φ_j is the non-compliance percentage for a limit state, and P_j is the penalty considered. The percentage of non-fulfillment is obtained from (acting resultant/strength –1). The design is checked at each iteration. The final result requires no penalties for convergence.

Table 1 Basic prices of the cost function of the reported vault structure

Unit	Unit cost (€)
m^3 of earth removal	3.010
m^3 of earth fill-in	4.810
m^2 of foundation formwork	9.015
m^2 of wall formwork	12.621
m^2 of upper vault formwork	21.035
m^3 of vault scaffolding	10.818
m^3 of lower slab concrete (labour)	3.606
m^3 of wall concrete (labour)	5.409
m^3 of upper vault concrete (labour)	4.508
m^3 of concrete pump rent	4.808
kg of steel B-500S	1.000
m^3 of concrete HA-25	43.724
m^3 of concrete HA-30	46.579
m^3 of concrete HA-35	49.434
m^3 of concrete HA-40	52.289
m^3 of concrete HA-45	55.144
m^3 of concrete HA-50	57.999

The analysis includes 49 discrete design variables that define the geometry, the grades of concrete and the reinforcement used. Variables include: a) five geometric values (the depth of the vault, the bottom and top depths of the lateral walls, the depths of the bottom foundation slab and its lateral toe), b) three different grades of concrete for the three types of elements, and c) 41 design variables to define the bar diameters, the spacing and the bar lengths of the reinforcement following a standard setup. The transverse and shear reinforcement variables considered in this study are provided in Fig. 1. A_1–A_8 are the basic internal and external reinforcement of the structure. A_9-A_{12} are the extra corner external reinforcement of the walls. A_{13} is the extra positive bending reinforcement of the arch. A_{14}-A_{17} and A_{20}-A_{23} are the shear reinforcement of the structure. A_{18} is the extra internal reinforcement in the bottom slab. Finally, A_{19} is the extra internal reinforcement of the arch. The variables were represented with 178 bits of binary code, representing an exorbitant number of possible solutions, given the resulting combinatorial explosion (on the order of 10^{53}).

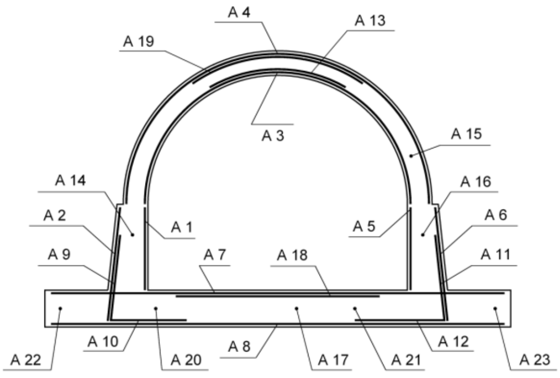

Figure 1 Reinforcement variables for the RC vaults.

The analytical parameters are fixed quantities and thus are not subject to optimization. They relate to geometric values, properties of the ground and the earth fill, partial safety coefficients and durability data. The main geometric parameters are the horizontal free span, which is twice the radius of the vault, the vertical height of the lateral walls and the earth cover. The main parameter of the ground is the stiffness modulus of the foundation. The data of the fill are its density and its internal friction angle. Durability exposure conditions are in accordance with the Spanish Concrete Code [11]. The design parameters for this RC vault are summarized in Table 2.

Table 2 Parameters of the reported vault structure

Parameter	Values
Horizontal free span	12.40 m
Vertical height of the lateral walls	3.00 m
Earth cover	1.00 m
Unit weight of the fill	20 kN/m^3
Internal friction angle of the fill	30º
Ballast coefficient of the ground	10 MN/m^3
Uniform distributed load	4 kN/m^2
Heavy vehicle	600 kN
Deflection of the free span limitation	1/250
Partial safety coefficient for permanent loading	1.60
Partial safety coefficient for life loading	1.60
ULS safety coefficient for concrete	1.50
ULS safety coefficient for steel	1.15
Spanish Concrete Code ambient exposure	IIa

Eq. (2) represents the constraints imposed by standard Spanish provisions to design this type of structure [11,12] and includes the verification of the ULSs of flexure and shear for the stress envelopes from traffic loads and earth fill. In this respect, the Spanish Concrete Code [11] is followed, except for deflections of the free span where a limitation of 1/250 for the quasi-permanent loading condition is established as generally recommended in the Eurocode 2 [4]. Permanent actions were self-weight, the weight of the earth cover and the active pressure of the landfill. Earth fill pressures were taken for partial filling heights of $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and total height, and three horizontal pressure coefficients of 0.20, 0.33 and 0.50 were considered. Additionally, a distributed load of 4.0 kN/m^2 and a heavy vehicle load of 600 kN [12] were taken into account. The stress and reactions were calculated as a structural model with six elements, eight nodes and 50 control sections, and the analysis was linear elastic under plane strain. Out-of-plane flexure moments had to be assumed as a practical one-fifth proportion of in-plane flexure moments. Furthermore, it was assumed that the bottom slab was supported by elastic springs whose stiffness is proportional to the ballast coefficient of the ground.

3 ITERATED LOCAL SEARCH BASED ON A GLOBAL BEST-DESCENT LOCAL ALGORITM

The ILS-GB search algorithm developed for this study is an ILS algorithm based on a global best (GB) descent. The ILS is a simple but powerful stochastic local search method that basically aims to avoid entrapments in poor local optima using a perturbation of the incumbent local optimum, originating a new intermediate solution, and then restarting the local search procedure from this modified solution. The perturbation is usually non-deterministic to avoid cycling. This procedure of alternating local searches and perturbation steps is repeated iteratively until a satisfactory criterion is met. The perturbation mechanism must not only be small enough so as to avoid a totally random restart point to exploit knowledge from previous iterations, but also large enough to escape the attractive basins around the local optimum. After a local search, the new local optimum can substitute the incumbent local optimum under a particular acceptance criterion. There are several possibilities, which range from never accepting the new local optimum unless there is an improvement and always accepting the new solution. Therefore, the key to ILS is that it focuses the search on a smaller subset defined by the local optima, where the perturbation operator performs a global random search. An essential principle of ILS is to exploit the trade-off between diversification – perturbation operator- and intensification –local search algorithm-. A thorough review of ILS algorithms was conducted by Lourenço et al. [18].

A general scheme of the ILS-GB framework proposed is depicted in Fig. 2. Our numerical experiments were conducted as follows. First, the algorithm starts with a randomly generated solution s. Then, a GB internal local search explores all neighbouring solutions and replaces s with the lowest cost neighbour $\hat{s}$. This $\hat{s}$ solution is the current record solution s^*. A perturbation operator reconstructs the current local optimum $\hat{s}$ and provides some intermediate solution s'. Next, the GB local search improves s' to another local optimum $\hat{s}$'. The best solution among the new local optimum $\hat{s}$' and the current record solution s^* decides which record solution is next. In this study, we adopted a random search acceptance criterion which always applies the perturbation to the most-recently visited local optimum. In addition, the ILS-GB algorithm stops after 100 iterations because, after several experiments, there is no significant improvement in the last local optimum.

```
Start
    s ← generate a random initial solution;
    ŝ ← GB local search (s);
    s* ← ŝ
    Repeat
        s' ← perturbation (ŝ);
        ŝ' ← GB local search (s');
        s* ← best of (ŝ', s*);
        ŝ ← s*;
    Until stop condition met
End
```

Figure 2 Pseudo-code for the ILS-GB method based on a random search criterion.

The GB local search algorithm developed in this study can be described as follows. The value of each variable is transformed into an integer by dividing its value into a multiple of quantity. This integer value is later transformed into a binary code, and a string of 178 bits of binary code can then be defined to represent all possible values from the 49 variables, and the algorithm recognises the maximum and minimum values for each of them. However, unlike its decimal numeric equivalent, binary coding is not homogeneous. For example, the number 15 is followed by 16 with a single digit change, but the corresponding binary codes, 01111 and 10000, requires five changes in digits to change from one to another. This is a well-known binary coding problem known as *Hamming Cliff.*

To avoid this problem, the well-known standard binary reflected Gray code can be used [2], which is an encoding of numbers so that adjacent numbers have a single digit differing by 1. This allows for an alternative representation by which the existing adjacency in the search space may be maintained in the space representing it. This GB local search consists in exploring, therefore, all solutions in which only one binary digit in the Gray code differs from the current solution. This is a quick, simple and generic movement which does not require any calibration.

As mentioned earlier, the perturbation operator partially destroys the current local optimum in a random manner. First, a number t of the variables are selected at random. Then, the chosen variables are modified by increasing or reducing their integer values by adding or subtracting an integer value given by Eq. (5):

$$x_{i+1} = x_i + x_{\max} \cdot X \tag{5}$$

where x_i is the current value of the i variable; x_{i+1} is the new value of this variable; $x_{\max}$ is the maximum integer value of the i variable, and X is a normally distributed random variable between -1 and 1, with mean $\mu = 0$. After several experiments, the number of variables selected at random was $t = 10$, and the standard deviation used for X was $\sigma^2 = 0.10$. To this end, the suggested perturbation operator follows the previously stated principle: the perturbation should not completely disrupt the structure of the current configuration.

Finally, there is another question to solve that has not been taken into consideration by the mechanisms explained so far. The number of times the ILS-GB algorithm is run should be large enough to ensure that the difference between the minimum value obtained from all runs and the theoretical global minimum is less than a specified threshold. Thus, a method based on the EVT was used and is related to that proposed by our research group for the optimization of building frames [25].

It is well-known that the three-parameter Weibull distribution belongs to the family of extreme value distributions, and represents the distribution of the smallest or largest values in random samples of increasing size. The Weibull cumulative distribution function can be expressed as follows:

$$F_X\ x_0 = \begin{cases} 1 - \exp\left\{ -\left(\dfrac{x_0 - \gamma}{\eta} \right)^{\beta} \right\}, & x_0 > \gamma \\ 0, & x_0 \le \gamma \end{cases} \tag{6}$$

where

$$\eta, \beta > 0 \tag{7}$$

where γ is the location parameter; η is the scale parameter, and β is the shape parameter.

Accordingly, if the statistical distribution of the best solutions found by ILS-GB fits a three-parameter Weibull distribution, then the estimated location parameter γ can be used as an estimation of the global optimum. However, estimating the γ parameter involves variability, since its value depends on the number of run solutions used to estimate it. To increase the significance of the estimator in question, the bootstrap method [9] is used to calculate the values of the minimum cost ($C_{\min}$), as well as the minimum ($\gamma_{\min}$) and the maximum ($\gamma_{\max}$) values of the location parameter of the samples taken of a certain size. If $\gamma_{\max} - \gamma_{\min}$ and $C_{\min} - \gamma_{\min}$ are lower than the two previously set limits, additional runs of the algorithm are not necessary.

4 RESULTS FROM COMPUTATIONAL EXPERIMENTS

In this Section, we examine the results from computational experiments involving ILS-GB optimization applied to a vault measuring 12.40 m in horizontal free span, considering the parameters defined in Table 1.The algorithm was coded in Fortran 95 with an Intel Fortran compiler 10.1. A personal computer with an Intel I7 processor with 2.94 GHz and 3 Gbyte RAM needed about 3.9 minutes to run the proposed ILS-GB algorithm (100 perturbations and 211,275 GB descent local search iterations on average).

The results and the histogram for the sample of 1000 minimal cost solutions found by ILS-GB are given in Fig. 3 and Fig. 4, respectively. The statistical description of this sample is as follows: the maximum and minimum values are €5676.62 and €5126.37, respectively; the sample mean value is €5398.33, with a confidence interval of ±€4.74 for a 0.05 level of significance; the standard deviation of the sample is €76.36; the median is €5392.08; the percentile of 5% is €5284.27. The distribution is leptokurtic (kurtosis coefficient of €0.386) and positively skewed (skewness coefficient of €0.335).

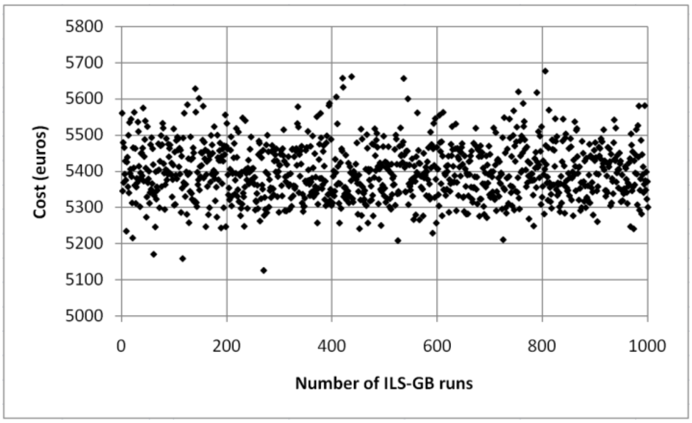

Figure 3 Cost results for 1000 ILS-GB runs.

To test the hypothesis that the 1000 results obtained by ILS-GB fit a three-parameter Weibull distribution function, one must verify that there is no reason to reject the null hypothesis that the histogram corresponds to a Weibull distribution; secondly, one must verify that the 1000 cost-optimized solutions found by ILS-GB are independent (see Fisher and Tippett [13]); lastly, the correlation coefficient of the Weibull distribution that best fits the 1000 numerical results must be high enough to provide a useful estimation.

Non-parametric tests such as Kolmogorov-Smirnov and Chi-squared statistics (see, e.g., Conover [7]) were computed assuming independence to assure that the ILS-GB solutions are Weibull-distributed. Since both statistics fell far below the critical value at 0.05 level of significance, there was no reason to reject the Weibull hypothesis.

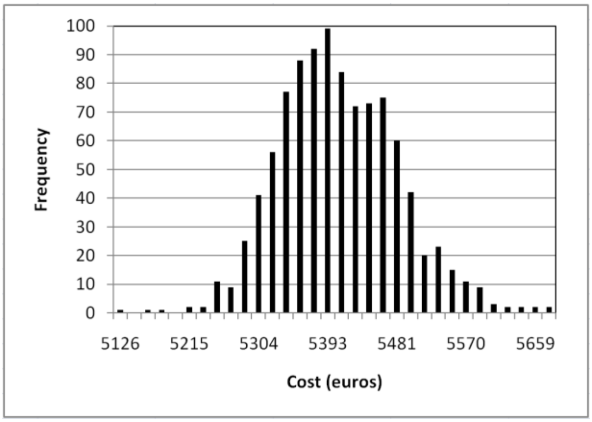

Figure 4 Histogram showing 1000 cost-optimized results.

One of the main assumptions underlying the use of EVT is that each ILS-GB solution is independent of the others and this is achieved by starting the ILS-GB algorithm search process from a random solution. A Wald-Wolfowitz run test was applied to the 1000 solutions obtained in order of occurrence so as to confirm that ILS-GB best solutions are independent. This is a non-parametric test which may be used to determine randomness in a sequence, and it is based on the total number of runs and the number of cases on the same side of a cut point (see, e.g., Conover [7]). In this case, there were 500 runs with respect to the median; the 2-tailed significance value was 0.255. Therefore, the run test does not offer any reason to reject the hypothesis of randomness.

Lastly, the parameters of the Weibull distribution that best fit the 1000 results obtained by ILS-GB were calculated, and this fit was quantified. To this end, ReliaSoft's Weibull++7 software [27] was used to estimate the three parameters of the Weibull distribution function. Two estimation methods were used: the maximum likelihood parameter estimation and the rank regression on Y estimation (according to the least squares principle, which minimizes the vertical distance between the data points and the probability density function). In our case, both estimates of the value were $\gamma =$ €5085.11 for the location parameter. This value is what the ILS-GB algorithm estimated to be provided for the global optimum of the problem using the EVT. The other parameters obtained for the regression on Y were $\eta = 341.7644$ and $\beta = 4.8507$. The Weibull fit had a correlation coefficient of $\varrho = 0.9857$, which was high enough for numerical results. The difference between the minimum value obtained after 1000 runs and the extreme value estimated was €41.26, a difference of just 0.81% compared to the theoretical minimum value. From the standpoint of structural engineering, this difference was small enough to accept the local optimum found by the proposed ILS-GB algorithm.

The ILS-GB algorithm had to be run enough times to assure that the difference between the minimum value found and the value estimated by the probability distribution was below a specified threshold. However, the γ parameter estimate tends to vary since it depends on the sample used. To analyse this, a confidence interval for a γ parameter was obtained using the method developed by

Paya et al. [25]. Nine samples were taken from the set of 1000 solutions with replacement of the sizes 10, 25, 50, 100, 250, 500 and 1000. For each sample the minimum cost value C_{min} was determined, and the γ parameter of the corresponding Weibull distribution was estimated. The minimum cost (C_{min}) is given in Fig. 5 along with the minimum (γ_{min}) and maximum (γ_{max}) values of the location parameter of the nine samples taken of each size.

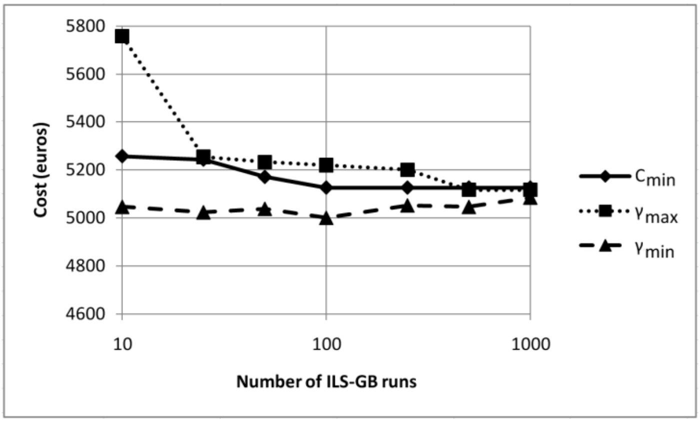

Figure 5 Value of the minimum costs (C_{min}) and the location parameter (γ) for nine samples with replacement for different numbers of ILS-GB algorithm runs.

As reflected in Table 3, the variability of the location parameter can be estimated using the difference between γ_{max} and γ_{min}. This range fell as the number of runs increased, and the relative difference with regard to γ_{min} dropped from 14.082% for 10 runs, to 0.637% for 1000 runs. This decrease was also observed in the case of the relative difference between the minimum cost and the γ_{min} parameter, dropping from 4.185% to 0.811% when the number of runs was increased from 10 to 1000, respectively.

Table 3 Minimum cost and estimated parameters for nine samples drawn with replacement of the set of 1000 runs

Number of tests	C_{min}	γ_{max}	γ_{min}	$(\gamma_{max} - \gamma_{min})/\gamma_{min}$ (%)	$(C_{min} - \gamma_{min})/\gamma_{min}$ (%)
10	5258.009	5757.456	5046.783	14.082%	4.185%
25	5243.628	5254.539	5024.220	4.584%	4.367%
50	5170.926	5233.396	5038.771	3.863%	2.623%
100	5126.371	5220.134	5001.432	4.373%	2.498%
250	5126.371	5201.192	5053.023	2.932%	1.452%
500	5126.371	5117.512	5046.660	1.404%	1.579%
1000	5126.371	5117.512	5085.107	0.637%	0.811%

As the number of local optima known depends on the runs carried out to estimate the parameters, the bootstrap [9] technique may be used. This technique is based on treating a random sample of n observations as if they were the entire population, from which new samples are taken by replacing the individual samples selected. The variability estimation for the location parameter was repeated with nine samples obtained through a random selection with replacement from among the set of local optima found. Fig. 6 illustrates the evolution of both the minimum cost and the γ_{max}

and γ_{min} parameters corresponding to the nine samples drawn using the bootstrap technique for 10, 25, 50, 100, 250 and 1000 runs.

The relative difference between γ_{max} and γ_{min} also diminished as the number of runs was increased, from 13.736% in the case of 10, to 0.637% in the case of 1000. The relative difference between the minimum cost and the estimated γ_{min} parameter dropped from 11.806% to 0.811%, when the GB algorithm was run from 10 to 1000 times. Here the increase from C_{min} and γ_{min} was stabalised to start with 1000 runs (see Table 4).

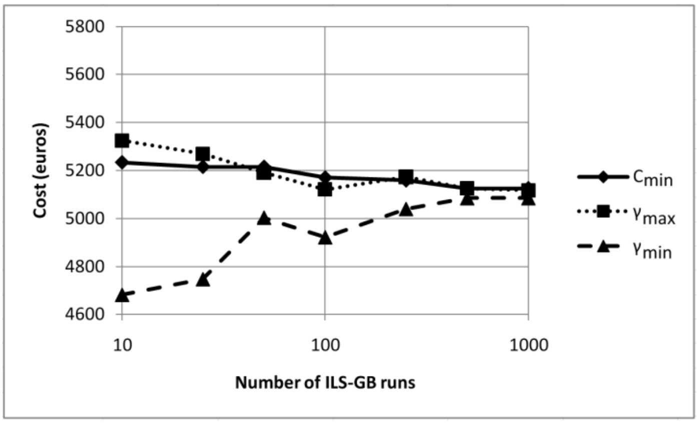

Figure 6 Value of the minimum costs (C_{min}) and the location parameters (γ) using the bootstrap technique for nine samples.

Table 4 Minimum cost and parameters estimated using the bootstrap technique for nine samples

Number of tests	C_{min}	γ_{max}	γ_{min}	$(\gamma_{max} - \gamma_{min})/\gamma_{min}$ (%)	$(C_{min} - \gamma_{min})/\gamma_{min}$ (%)
10	5234.740	5325.131	4682.005	13.736%	11.806%
25	5215.935	5269.513	4747.440	10.997%	9.868%
50	5215.935	5191.065	5004.258	3.733%	4.230%
100	5170.926	5121.616	4922.380	4.048%	5.049%
250	5159.103	5173.776	5040.125	2.652%	2.361%
500	5126.371	5126.371	5085.107	0.811%	0.811%
1000	5126.371	5117.512	5085.107	0.637%	0.811%

Thus, an objective stopping criterion was established for a multi-start algorithm based on the local GB search. Starting from a random solution, a local search was applied until a minimum cost was achieved. With different starts, a sample of local optima was obtained using the bootstrap technique, which allowed nine samples to be drawn to determine 1) the difference between the minimum cost reached up to a certain time and the theoretical minimum estimated by means of a Weibull distribution as well as 2) the difference between the maximum and minimum value of the γ parameters estimated. The multi-start algorithm was stopped when neither the difference between the minimum and the theoretical solution found nor the variability of the location parameters exceeded a certain threshold. We assumed that the variability in the determination of the location parameter γ and the difference between the minimum and theoretical cost reached were lower than 1%. Thus interpolating the data in Table 3, 339 runs would be required. Fig. 7 and Table 5 show the variation of the cumulative mean and the standard deviation of the cost for 1000 runs of the ILS-GB. All this indicates that between 250 and 500 runs are sufficient to stabilize the results for the ILS-GB algorithm. Moreover, an approximate 95% confidence interval of the population mean for 339 runs is estimated with an error less than 9 euros (Table 5). This euro value is 0.155% of the average cost solution for 339 ILS-GA runs, which is low enough to be acceptable.

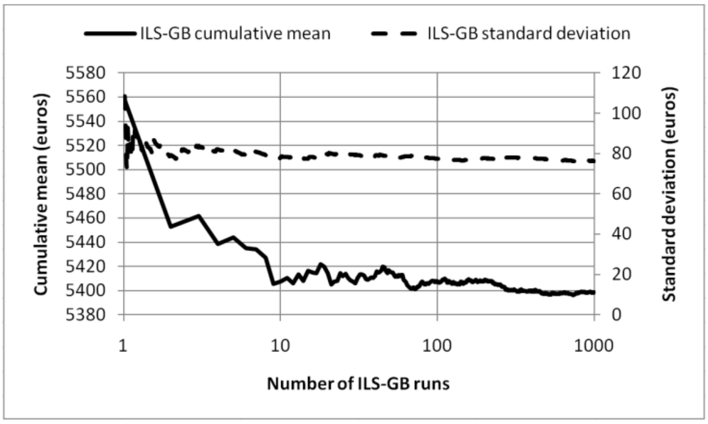

Figure 7 Cumulative mean and standard deviation versus number of ILS-GB runs.

Table 5 Results of the ILS-GA algorithm

Number of runs	Minumum cost (€)	Average cost (€)	Standard deviation (€)	Estimated error (€)	Estimated error/ average cost (%)
10	5234.740	5407.637	87.647	62.699	1.159
25	5215.935	5412.153	90.330	37.286	0.689
50	5215.935	5415.817	84.458	24.003	0.443
100	5170.926	5407.103	79.313	15.738	0.291
250	5159.103	5405.153	79.849	9.946	0.184
339	5126.371	5400.608	78.394	8.375	0.155
500	5126.371	5397.541	79.228	6.961	0.129
1000	5126.371	5398.325	76.369	4.738	0.088

The cost of the best ILS-GB solution is 5126.371 euros/m, which considering the basic prices listed in Table 2, is 7.72% less than the cost of the vault designed in 2002 by the third author for the Valle Romano motorway in Malaga (Spain) following standard design office procedures. The top vault is only 0.25 m deep for the 12.40 m span. The depths of the top and bottom of the lateral walls are 0.40 m and 0.50 m, respectively. The bottom slab is 0.70 m depth and the toe 2.60 m. 25 MPa concrete is used for the bottom slab, 30 MPa concrete for the top vault, and 35 MPa concrete for the lateral walls. The reinforcement setup is summarized in Fig. 8. This best ILS-GB solution costs 3.11% more than the best minimum cost reported by Carbonell et al. [3]. However, the variable action taken into account in [3] was only a superficial embankment uniform load of 10 kN/m^2, whereas a distributed load of 4.0 kN/m^2 as well as a heavy vehicle load of 600 kN [3] were considered in the present study. In addition, the meta-SA algorithm used by Carbonell et al. [3] required approximately 24 h of computing time to calibrate the SA, while the ILS-GB algorithm developed in the present research does not require calibrating.

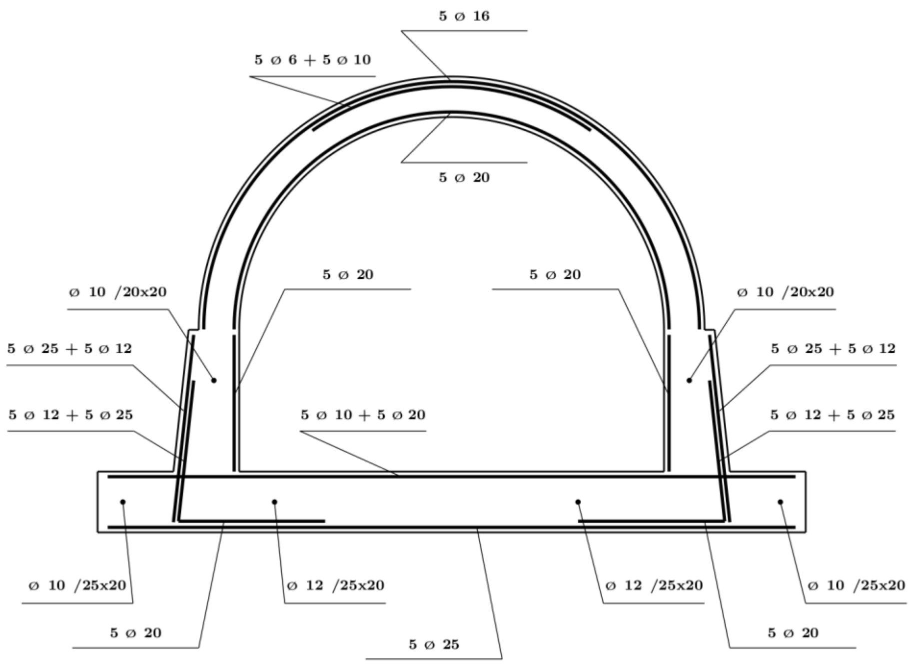

Figure 8 Reinforcement for the ILS-GB best result, referred to a length of 1 m vault.

5 CONCLUSIONS

In this paper we describe an algorithm which is useful for the automatic design as well as cost minimization of RC vaults based on an iterated local search and a Gray code global best-descent local search named ILS-GB. This ILS-GB algorithm combines an iterated local search strategy, a global best local search and a random search acceptance criterion which always applies a perturbation to the most-recently visited local minimum. This algorithm does not require feasible solutions as initial solutions or calibration. The local optima found by this ILS-GB are extreme values forming a simple random sample fitting a Weibull distribution of three parameters, γ being an estimate of the global optimum which this algorithm could reach. The best value obtained by ILS-GB differed only 0.81% compared to the theoretical minimum value. The study verified two objective stopping criterion for a multi-start ILS-GB algorithm: 1) the difference between the minimum cost found and the γ parameter and 2) the confidence interval for this parameter are limited, e.g., to 1%. The parameters for a vault are estimated from nine samples taken with the bootstrap technique. The results are quite encouraging and suggest that this approach may easily be adapted to other optimization problems. Moreover, the optimization results indicate a savings of about 7% compared to a traditional design. It is worth noting that the results obtained may certainly be improved, e.g., by extending to population-based ILS, which seems a worthy subject for future research.

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