IALCCE’08 Symposium, Varenna

Cost versus Sustainability of Reinforced Concrete Building Frames by Multiobjective Optimization

I. Payá-Zaforteza
V. Yepes
F. González-Vidosa
A. Hospitaler
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1. Introduction.

Reinforced concrete building frames

RC frame = columns + beams
1. introduction.

- traditional approach to design:

   1. initial design
   2. structural analysis
   3. modified design
   4. ¿O.K.?
      - no
      - yes
   5. final design

prof. Fernández Casado, one of the most important structural engineers of the Spanish tradition
1. introduction.

- alternative approach:

OPTIMAL STRUCTURE: minimum cost

... multiobjective optimization (cost, sustainability, construction schedules...)

Amazonic jungle

train station (Berlin Hauptbahnhof) under construction
objective: to develop an algorithm that is able to find the optimal RC frame when considering both, economic cost and sustainability.
1. introduction.

case study:

Horizontal and vertical loads according to Spanish codes

(6 elementary load cases and 48 different load combinations)
2. optimization problem definition.

- **general statement** of the optimization problem:

  optimize $\mathbf{f} = (f_1(\mathbf{X}), \ldots, f_n(\mathbf{X}))$

  $\mathbf{X} = (x_1, \ldots, x_n)$

  $\mathbf{P} = (p_1, \ldots, p_m)$

  satisfying: $g_i(\mathbf{X}, \mathbf{P}) \leq 0 \quad i = 1, \ldots, k$

  feasible solutions vs. non feasible solutions

  vector of objective functions

  design variables

  parameters

  constraints
2. optimization problem definition.

2.1 objective functions.

- economic cost, $C$:

$$C = \sum_{\text{columns}} (C_{\text{STEEL}} + C_{\text{FORMWORK}} + C_{\text{CONCRETE}}) + \sum_{\text{beams}} (C_{\text{SCAFFOLDING}} + C_{\text{FORMWORK}} + C_{\text{STEEL}} + C_{\text{CONCRETE}})$$

where $C_i = p_i \times m_i$,

$p_i$: construction unit price
$m_i$: quantities of construction units

**Construction unit prices, $p_i$**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Description</th>
<th>Cost (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>kg</td>
<td>Steel B-400</td>
<td>1.27</td>
</tr>
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<td>kg</td>
<td>Steel B-500</td>
<td>1.30</td>
</tr>
<tr>
<td>m³</td>
<td>Concrete C-25</td>
<td>78.40</td>
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<tr>
<td>m³</td>
<td>Concrete C-30</td>
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<td>m³</td>
<td>Concrete C-35</td>
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<td>m³</td>
<td>Concrete C-40</td>
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<td>m³</td>
<td>Concrete C-45</td>
<td>112.13</td>
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<tr>
<td>m³</td>
<td>Concrete C-50</td>
<td>118.60</td>
</tr>
<tr>
<td>m²</td>
<td>Formwork in beams</td>
<td>25.05</td>
</tr>
<tr>
<td>m²</td>
<td>Formwork in columns</td>
<td>22.75</td>
</tr>
<tr>
<td>m²</td>
<td>Scaffolding for beams</td>
<td>38.89</td>
</tr>
</tbody>
</table>
2. optimization problem definition.

2.1 objective functions.

- environmental cost, EC :

\[
EC = \sum_{\text{columns}} (EC_{\text{STEEL}} + EC_{\text{CONCRETE}}) + \sum_{\text{beams}} (EC_{\text{STEEL}} + EC_{\text{CONCRETE}})
\]

where \( EC_i = d_i \times m_i \),

\( d_i \): material environmental cost

\( m_i \): quantities of materials

<table>
<thead>
<tr>
<th>Unit</th>
<th>Description</th>
<th>Environmental Cost</th>
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</thead>
<tbody>
<tr>
<td>kg</td>
<td>Steel B-400</td>
<td>61</td>
</tr>
<tr>
<td>kg</td>
<td>Steel B-500</td>
<td>61</td>
</tr>
<tr>
<td>m³</td>
<td>Concrete C-25</td>
<td>39100</td>
</tr>
<tr>
<td>m³</td>
<td>Concrete C-30</td>
<td>41400</td>
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<td>m³</td>
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<tr>
<td>m³</td>
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<td>m³</td>
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<tr>
<td>m³</td>
<td>Concrete C-50</td>
<td>61300</td>
</tr>
</tbody>
</table>
2. optimization problem definition.

2.2. design variables.

115 variables:
- steel reinforcement strength
- 6 concrete grades
- 36 cross sections of beams and columns
- 72 types of steel reinforcement

(all of them are discrete)

beams reinforcement setup

longitudinal reinforcement

left

central

right

shear reinforcement

extra top reinf. (left)
extra top reinf. (right)

2. optimization problem definition.

2.3. structural constraints.

- Ultimate Limit States (ULS): flexure, shear, buckling.
- Service Limit States (SLS): deflections, cracking.
- Drifts.
- Constructability: e.g. minimum and maximum distances among reinforcement bars, ties spacing...
3. optimization procedure.

The two objective functions considered are in opposition.

PARETO SET OF SOLUTIONS

Pareto Set of Solutions: 1, 3, 4
3. optimization procedure.

Suppapitnarm Multiobjective Simulated Annealing, SMOSA (2000)

based on the analogy of crystal formation from masses melted at high temperatures and let cool slowly
3. optimization procedure.

SMOSA.

1) Creation of an initial random solution $S_1$.

2) Creation of a new solution $S_2$ (through a small random move of the values of some of the variables that define $S_1$).

3) If $S_2$ is Pareto Optimal + $S_2$ is feasible
   $S_2$ replaces $S_1$ as working solution
   $S_2$ becomes a member of the Pareto Set

   If $S_2$ is not Pareto Optimal + $S_2$ is feasible
   $S_2$ replaces $S_1$ with a probability:
   \[ p = \frac{1}{e^{\Delta f_i / t_i} e^{\Delta f_j / t_j}} \]
   \[ \Delta f_i = f_i(S_2) - f_i(S_1) \] difference in the objective function values.

   “$t_i$” temperature for “$i$” objective, it is reduced every “$L_M$” iterations
   ($L_M$ : Markov chain)

4) back to 2) until a stop criterion is satisfied.
4. results.

software programmed in Fortran

calibration of the algorithm: $L_M=135000$ ; $r = 0.8$ ; move up to 3 variables.

CPU computing time: 80 min in a Pentium IV of 3.2 GHz.

![Pareto Set of solutions](chart)

*Highest sustainability* 26.7%

*Lowest cost* 4%
4. results.

lowest environmental cost solution

Beam results

<table>
<thead>
<tr>
<th>Name</th>
<th>Dimensions</th>
<th>Top reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Depth (cm)</td>
<td>Width (cm)</td>
</tr>
<tr>
<td>B-1</td>
<td>48</td>
<td>20</td>
</tr>
<tr>
<td>B-2</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>B-3</td>
<td>48</td>
<td>20</td>
</tr>
<tr>
<td>B-4</td>
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<td>B-5</td>
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<td>20</td>
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<tr>
<td>B-6</td>
<td>47</td>
<td>20</td>
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</table>

Column results

<table>
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<tr>
<th>Name</th>
<th>Dimensions</th>
<th>Long. Rein.</th>
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<tbody>
<tr>
<td></td>
<td>a (cm)</td>
<td>b (cm)</td>
</tr>
<tr>
<td>C-1</td>
<td>35</td>
<td>25</td>
</tr>
<tr>
<td>C-2</td>
<td>35</td>
<td>25</td>
</tr>
<tr>
<td>C-3</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
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<td>C-7</td>
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<tr>
<td>C-8</td>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td>C-9</td>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td>C-10</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>C-11</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>C-12</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

Concrete and steel results

Characteristic concrete compressive strength, fck (28 days)
RC elements from ground to 4th floor: 35 MPa
RC elements 5th and 6th floors: 25 MPa

Characteristic reinforcement yield strength, fyk: 500 MPa
5. conclusions.

this work shows:

- the capability of SMOSA for the optimization of cost and sustainability of complex RC structures.

- more sustainable solutions than the lower cost solutions are available at a cost increment acceptable in practice.

Thank you for your attention!
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