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A model of resource management in small cells with dynamic traffic and backhaul constraints

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Abstract—**During the last years, the volume of data-traffic carried by mobile cellular networks has experienced a major growth and progress. The current networks' features are not enough to face this development paradigm and the novel concept of small cells has emerged to increase the network capacity. However, the deployment of small cells introduces several technical challenges such as the cross-tier interference between the macrocell and the small cells or the use of the land-line to send the backhaul data. In this paper, an analytical model is proposed to study the impact of the resource management on the small cell performance when the characteristics of the traffic vary dynamically over time. This model is applied in order to find the admission control policy which maximizes the throughput of the small cell users depending on the backhaul capacity variations and the interference produced by the users connected to the macrocell.**

I. Introduction

During the last years, there has been an unprecedented growth and progress in the data-traffic volume carried by cellular networks. This increasing popularity will continue to grow as mobile systems are expected to support a larger variety of multimedia services. Moreover, according to recent surveys [1], the traffic which is expected to produce the bulk of the network load will mainly occur indoor. Unfortunately, the current networks' features are not enough to face this development paradigm. In this context, the novel concept of small cells [2], [3] has emerged as a solution to increase both network capacity and indoor coverage.

Small cells provide service to small geographical areas and require low-power base stations called Small cell Access Points (SAPs). These SAPs are owned and installed by the users and used to provide indoor service with a benefit for users and operators. Users improve their QoS, while operators can manage the growth of traffic without the need to develop new network infrastructure. Moreover, the SAPs send the backhaul data to the cellular operator network over the land-line, thus allowing operators to release resources for other users connected to the macrocell.

From the perspective of the small cell connectivity priority, two types of cellular users are defined: i) *Small cell Users* (SUs) which are registered in the small cell and can always connect to the small cell; ii) *Macrocell Users* (MUs) which are not registered in the small cell. In addition, one more type of user is defined due to the use of the land-line as data backhaul: *Land-line Users* (LUs), which are non-cellular users that generate traffic carried by the same land-line connection that the SAP uses as a backhaul.

The deployment of small cells introduces several technical challenges [4]. One of the performance-limiting factors is the cross-tier interference between the macrocell and the small cell [5]. This problem has been widely addressed in the literature and many approaches have been proposed, which involve the use of power control [6] or advanced spectrum management techniques [7]. Moreover, the radio interference can be managed by allowing strong macrocell interferers to connect and acquire some level of service in the small cells [8].

The access of users to the small cell is regulated by an Admission Control (AC) policy. This mechanism provides different levels of priority to SUs and MUs. Three basic AC modes have been defined [8]: i) *Closed access*, only SUs can access the small cell; ii) *Open access*, all users can make use of the small cell resources and iii) *hybrid access*, a limited amount of the small cell resources are available to all users, while the rest of resources are only available to SUs.

To the best of our knowledge, so far the small cell architecture has been studied considering that the temporal characteristics of traffic are static. In this work, we consider that users generate finite flows that come and go. As a result, the load of the system varies dynamically over time. Our next step will be to consider a scenario where the number of users and their positions can also vary over time. Moreover, in previous studies, the bandwidth limitations introduced by the backhaul are not given sufficient attention. In this work, we also consider the temporal variation and the limitations of the backhaul capacity. The major contributions of this work are, then, the introduction of dynamic traffic and the consideration of the backhaul limitations. In this paper, we propose an analytical model to study the impact of the temporal traffic dynamics and the backhaul constraints on the small cell performance. This study allows to evaluate the performance of different AC policies and thus, to decide which AC mode is more appropriate for a given scenario.

This paper is organized as follows. In Section II, we describe the interference model proposed. In Section III, we describe first the backhaul traffic model and then we present the traffic model of the system. In Section IV, we discuss the numerical results. Finally, Section V concludes the paper.

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II. Interference Model

In this section, we present the interference model which will be applied in Section III-B to study the impact of the interference on the performance analysis.

We analyze the performance of a small cell which is inside a macrocell. Although for our purposes it is not of great importance which traffic direction is considered, we consider only the uplink direction. We consider that an MU can be in different geographical regions within the macrocell according to the amount of interference caused to the SAP. We consider K regions denoted by A_i, $i = 1, 2, \cdots, K$, where region A_1 is the region where the MUs connected to the BS cause the strongest interference and A_k is the region where the MUs connected to the BS cause the weakest interference. An MU in A_i reduces the achievable bitrate in the small cell by a fraction γ_i ($1 > \gamma_1 > \cdots > \gamma_K$) due to the interference that the MU causes to the SAP.

Let d be the distance between the BS and an MU connected to the BS, and d' the distance between this MU and the SAP; see Fig 1. When the distance d is large, the MU needs more power to transmit and the interference is higher. When the distance d' is short, the MU produces higher interference since it is closer to the SAP. Given that one of the most dominant factors in the reduction of the transmitted signal power is the path-loss, the SIR achieved by a user connected to the small cell is lower for a large d and a short d', i.e.:

$$SIR \propto \left(\frac{d'}{d}\right)^{\zeta}, \tag{1}$$

where ζ is the path-loss exponent. Then, the thresholds $\delta_1 > \delta_2 > \cdots > \delta_{K-1}$ divide the macrocell into zones as:

$$\begin{aligned} &\text{zone } 1 \quad \text{if} \quad \delta_1 \le \frac{d}{d'} \\ &\text{zone } i \quad \text{if} \quad \delta_i \le \frac{d}{d'} \le \delta_{i-1} \\ &\qquad \vdots \\ &\text{zone } K \quad \text{if} \quad \frac{d}{d'} \le \delta_{K-1}. \end{aligned} \tag{2}$$

The border between region i and region $i+1$ is given by the points where $d/d' = \delta_i$, i.e. by the curve given by the points where the quotient of the distances d and d' is constant. Without loss of generality, in order to obtain this curve we consider that the BS and the SAP are in the abscissa axis symmetrically situated respect to the origin. If the distance between the BS and the SAP is D, the BS is at $(-\frac{D}{2}, 0)$ and the SAP at $(\frac{D}{2}, 0)$. Then, we are searching the point (x, y) where:

$$\frac{\sqrt{(x+\frac{D}{2})^2 + y^2}}{\sqrt{(x-\frac{D}{2})^2 + y^2}} = \delta_i. \tag{3}$$

Further manipulation of this expression results in

$$\left(x - \frac{\frac{D}{2}(\delta_i^2+1)}{\delta_i^2 - 1}\right)^2 + y^2 = \frac{4(\frac{D}{2})^2\delta_i^2}{(\delta_i^2-1)^2}, \tag{4}$$

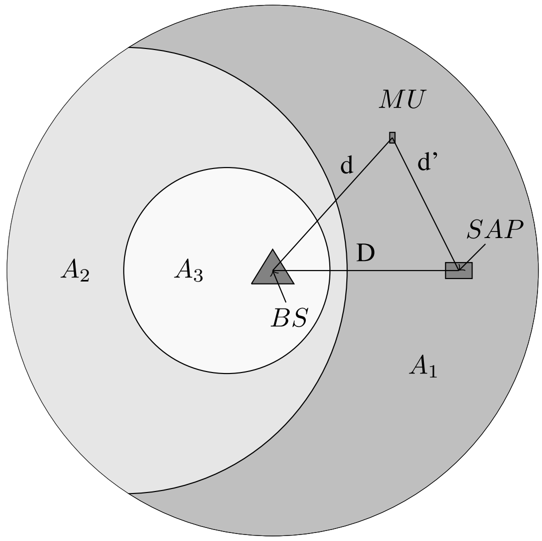

Fig. 1. Different interference regions, $K = 3$.

which is a circumference with center:

$$\left(\frac{\frac{D}{2}(\delta_i^2+1)}{\delta_i^2-1}, 0\right), \tag{5}$$

and radius

$$\frac{2\frac{D}{2}\delta_i}{|\delta_i^2 - 1|}. \tag{6}$$

Notice that if $\delta_i = 1$, the curve is the perpendicular bisector of the segment which connects the BS and the SAP points, i.e., it is the ordinate axis, $x = 0$. In Figure 1, an example with $K = 3$ regions with $D = 70$, $\delta_1 = 2/3$ and $\delta_2 = 4/9$ is shown.

We consider that the MUs can be associated to the BS or to the SAP. These users can be idle or active, i.e. they can be idle or uploading a data flow. If an idle MU associated to the BS becomes active, it will be served by the BS. If an idle MU associated to the SAP becomes active, it will be served by the SAP. In this work, we study which MUs should be associated to the SAP, in order to obtain the best small cell performance.

We define H as the number of SUs, which are registered to the SAP. Thus, they are always associated to the SAP. Let M_i be the total number of MUs in A_i and let $M_i' \le M_i$ be the number of MUs in A_i which are associated to the SAP. These users can be uploading a data flow or not.

Henceforth, we consider three interference regions, i.e. $K = 3$. The MUs in A_1 and A_2 are considered to be close enough to the SAP to be handed over from the BS to the small cell, while those in A_3 are not, i.e. $M_3' = 0$. We also consider that MUs in zone A_3 do not produce interference, i.e. $\gamma_3 = 0$. Obviously, if there are still users in A_1, users in A_2 are not handed over from the BS to the SAP, i.e. if $M_1' < M_1$ then $M_2' = 0$. The number of users associated to the BS, N_m, and the SAP, N_s, are given by

$$\begin{aligned} N_m &= M_1 + M_2 + M_3 - M_1' - M_2', \\ N_s &= H + M_1' + M_2'. \end{aligned}$$

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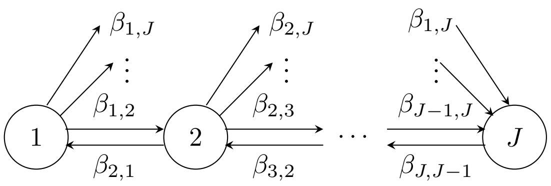

Fig. 2. State transitions of the CTMC which models the backhaul.

III. TRAFFIC MODEL

In this section, we present the traffic model proposed. First, we describe the backhaul traffic model and then, the system model, which considers the interference and the backhaul limitations, is proposed.

A. Backhaul traffic model

The traffic going through the SAP competes with the LUs' traffic for the wireline bandwidth. We assume that priority is given to LUs' traffic since it is the primary reason for the Internet access service subscription. It is assumed that there is a minimum bitrate that the SAPs would be able to get at anytime in order to prevent starvation and also to guarantee that the SUs can get access to the voice service at anytime. This is required as the SUs may not have land-line telephone subscription nor a sufficiently good coverage from the BS.

The load generated by the LUs is modeled by a finite-state Continuous Time Markov Chain (CTMC) with J states. The transition rate from state j to state j' is denoted by $\beta_{j,j'}$ (see Figure 2). When the CTMC is in state j, $j = 1, 2, \cdots, J$, the available bitrate in the backhaul is C_j, where $C_1 > C_2 > \cdots > C_J > 0$. The minimum bitrate reserved to the SAP in order to avoid starvation is given by C_J. The Additive-Increase Multiplicative-Decrease (AIMD) algorithm used in TCP is emulated by the backhaul dynamics and a slow-bandwidth acquisition strategy is used to avoid sending more data than the backhaul is capable of transmitting. Thus, transitions to states with worse bitrates are possible to any state with $j' > j$, while transitions to states with better bitrates only can happen to states with $j' = j - 1$.

Each C_j could be associated with several states. This will allow to model more complex behaviors and the sojourn time at each load condition will not be restricted to be exponentially distributed.

B. System model

Each user (MU or SU) can be either idle or uploading an elastic flow, which corresponds to transfer of digital documents. The model could be extended to the case of real-time services. A user is idle for a random time, which is *iid* across users, until a flow is generated for upload. The lengths (in bits) of the flows generated by all users form a sequence of *iid* random variables. The time a session spends uploading a flow will depend on the length of the flow and the amount of available resources. For the sake of mathematical tractability we assume that the duration of an idle period is exponentially distributed with mean $1/\alpha$. Likewise, the length of a flow is also exponential with mean σ.

A fixed number of users is considered since we assume that the variation of number of users and their locations occur at a much longer time scale than the flow dynamics. Decision regarding handovers for MUs nearby the SAP are also supposed to be done at a longer time scale.

We model the proposed system using a multidimensional CTMC. The states of this CTMC modeling the whole system are represented by the state vector $\boldsymbol{s} = (x, y, z_1, z_2, z_3)$ where $x \leq N_s$ denotes the number of active users connected to the SAP regardless of whether they are SUs or MUs; $y = 1, 2, \cdots, J$ denotes the state of the backhaul with the corresponding available bitrate C_y; and $z_i \leq M_i - M_i'$ denotes the number of active MUs in zone A_i which are connected to the BS.

The maximum bitrate in the macrocell (small cell) that could be achieved by employing all the macrocell (small cell) resources and assuming a sufficiently high SINR is R_m (R_s). Due to impatience of users and hardware limitations of terminals the bitrate of users is restricted to be in the interval $[r_m, r_M]$. The throughput obtained by each active MU connected to the BS at state $\boldsymbol{s}$, $\phi_m(\boldsymbol{s})$, is given by:

$$\phi_m(\boldsymbol{s}) = \min\left\{r_M, \frac{R_m}{z_1 + z_2 + z_3}\right\}. \tag{7}$$

We assume a fair allocation of resources among users. Then, the average fraction of time, η_t, that each active user connected to the BS is allocated is given by:

$$\eta_t = \frac{\phi_m(\boldsymbol{s})}{R_m} = \min\left\{\frac{r_M}{R_m}, \frac{1}{z_1 + z_2 + z_3}\right\}. \tag{8}$$

We consider that the throughput is reduced due to interference by the fraction $\eta_t \cdot (\gamma_1 z_1 + \gamma_2 z_2)$ and then, the throughput obtained by each active user connected to the SAP at state $\boldsymbol{s}$ with $x > 0$, $\phi_s(\boldsymbol{s})$, is given by:

$$\phi_s(\boldsymbol{s}) = \min\left\{r_M, \frac{R_s}{x}\left[1 - \eta_t \cdot (\gamma_1 z_1 + \gamma_2 z_2)\right]^+, \frac{C_y}{x}\right\}. \tag{9}$$

We consider a non-preemptive AC policy at the flow level. When a flow is not accepted, it is lost, i.e. there are neither queuing nor retrials. A flow from an SU or an MU associated to the SAP is accepted to the small cell if, after acceptance, all ongoing flows in the small cell obtain a bitrate equal or bigger than the minimum, r_m. We denote by $a_s(\boldsymbol{s})$ the probability of accepting a flow in the small cell in state $\boldsymbol{s}$. We denote by $a_{m,i}(\boldsymbol{s})$ the probability of accepting a flow in zone i of the macrocell in state $\boldsymbol{s}$. Thus, if $\boldsymbol{e}_k$ is a 5-dimensional vector with a 1 on the k-th position and 0's elsewhere, the probabilities of acceptance $a_s(\boldsymbol{s})$ and $a_{m,i}(\boldsymbol{s})$ can be determined as follows:

$$a_s(\boldsymbol{s}) = \begin{cases} 1 & \text{if} \quad \phi_s(\boldsymbol{s} + \boldsymbol{e}_1) \geq r_m \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$a_{m,i}(\boldsymbol{s}) = \begin{cases} 1 & \text{if} \quad \phi_m(\boldsymbol{s} + \boldsymbol{e}_{2+i}) \geq r_m \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

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Notice that the AC policy of the macrocell does not consider the users in the small cell. Therefore, if an MU which is associated to the BS becomes active and it is accepted, it can happen that the bitrate of the users connected to the SAP becomes lower than the minimum r_m due to interference. Similarly, the backhaul variations cannot be controlled by the AC policy of the small cell. Therefore, if the capacity of the backhaul drops, it can happen that the bitrates of the users connected to the SAP becomes lower than the minimum. In these two cases, all users connected to the SAP will be served with a low QoS.

The system has a finite user population with N_s users associated to the SAP, $M_1 - M_1'$ MUs in zone A_1 associated to the BS, $M_2 - M_2'$ MUs in zone A_2 associated to the BS and M_3 MUs in zone A_3 associated to the BS. Thus, from the above, the transition rates from state $\boldsymbol{s}$ to state $\boldsymbol{s}'$, denoted by $q_{\boldsymbol{s},\boldsymbol{s}'}$, can be readily derived as:

$$q_{\boldsymbol{s},\boldsymbol{s}'} = \begin{cases} a_s(\boldsymbol{s})(N_s - x)\alpha & \text{if } x \to x+1, \\ x\phi(\boldsymbol{s})/\sigma & \text{if } x \to x-1, \\ \beta_{y,y'} & \text{if } y \to y', \\ a_{m,1}(\boldsymbol{s})(M_1 - M_1' - z_1)\alpha & \text{if } z_1 \to z_1+1, \\ z_1\phi(\boldsymbol{s})/\sigma & \text{if } z_1 \to z_1-1, \\ a_{m,2}(\boldsymbol{s})(M_2 - M_2' - z_2)\alpha & \text{if } z_2 \to z_2+1, \\ z_2\phi(\boldsymbol{s})/\sigma & \text{if } z_2 \to z_2-1, \\ a_{m,3}(\boldsymbol{s})(M_3 - z_3)\alpha & \text{if } z_3 \to z_3+1, \\ z_3\phi(\boldsymbol{s})/\sigma & \text{if } z_3 \to z_3-1, \\ 0 & \text{otherwise.} \end{cases}$$

Recall that $\beta_{y,y'}$ are the transition rates due to variations of the backhaul bitrate. They are given by the CTMC which models the backhaul capacity represented in Fig. 2.

The size of the state space is:

$$|\mathcal{S}| = X \cdot Y \cdot Z_1 \cdot Z_2 \cdot Z_3,$$

where $X = N_s + 1$, $Y = J$, $Z_1 = M_1 - M_1' + 1$, $Z_2 = M_2 - M_2' + 1$ and $Z_3 = M_3 + 1$.

The set of feasible states is thus given by

$$\mathcal{S} := \{\boldsymbol{s} : \quad x, y, z_i \in \mathbb{N}; \quad x < X \\ y < Y; \quad z_i < Z_i\}. \tag{12}$$

Let $\mathcal{L}$ be the set of states where the active users connected to the SAP are served with a bitrate lower than the minimum,

$$\mathcal{L} := \{\boldsymbol{s} \in \mathcal{S}; \quad \phi_s(\boldsymbol{s}) < r_m\}. \tag{13}$$

Let $\boldsymbol{\pi}$ denote the vector of stationary probabilities which is obtained by solving the global balance equations together with the normalization equation,

$$\pi(\boldsymbol{s}) \sum_{\forall \boldsymbol{s} \neq \boldsymbol{s}'} q_{\boldsymbol{s},\boldsymbol{s}'} = \sum_{\forall \boldsymbol{s}' \neq \boldsymbol{s}} q_{\boldsymbol{s}',\boldsymbol{s}} \pi(\boldsymbol{s}'); \quad \sum_{\forall \boldsymbol{s}} \pi(\boldsymbol{s}) = 1. \tag{14}$$

The average bitrate obtained by the users associated to the SAP, $E[\phi_s]$, is thus given by:

$$E[\phi_s] = \frac{\sum_{\boldsymbol{s} \in \mathcal{S}} x\phi_s(\boldsymbol{s})\pi(\boldsymbol{s})}{\sum_{\boldsymbol{s} \in \mathcal{S}} x\pi(\boldsymbol{s})}. \tag{15}$$

Let us denote by P_b the blocking probability in the small cell, which is the probability that a flow is not accepted in the small cell and therefore it is lost. Then:

$$P_b = \sum_{\boldsymbol{s} \in \mathcal{S}, x < N_s} (1 - a_s(\boldsymbol{s}))\pi(\boldsymbol{s}). \tag{16}$$

The probability that the active users connected to the SAP are served with a bitrate lower than the minimum r_m and therefore with low QoS, P_{QoS}, is:

$$P_{QoS} = \sum_{\boldsymbol{s} \in \mathcal{L}} \pi(\boldsymbol{s}). \tag{17}$$

IV. Numerical evaluation

In this section, we present a set of experiments of some specific scenarios in order to demonstrate the feasibility of the model.

We consider a system with $K = 3$ interference regions with $D = 70$, $\delta_1 = 2/3$ and $\delta_2 = 4/9$. The achievable bitrate is reduced by a fraction $\gamma_1 = 0.8$ and $\gamma_2 = 0.4$ for a MU connected to the BS in A_1 and A_2, respectively.

The number of users considered in the system are $H = 10$ SUs, $M_3 = 2$, $M_3' = 0$ and $M_2 = 15 - M_1$. The system parameter values, unless otherwise specified, are: $R_m = 5$ Mbps, $R_s = 1$ Mbps, $r_M = 1$ Mbps, $r_m = 10$ kbps, $\alpha = 100$ s^{-1}, $\sigma = 7$ kbits.

Regarding the backhaul, we consider three backhaul states, i.e. $J = 3$, which can be given a qualitative interpretation as follows: i) $y = 1$, light traffic where $C_1 = 1$ Mbps; ii) $y = 2$, medium traffic where $C_2 = 0.5$ Mbps ; iii) $y = 3$, heavy traffic where $C_3 = 100$ kbps. In Table I, the value of the transition rate matrices is shown for three different backhaul scenarios. In Scenario 1 the LU traffic is mostly light. In Scenario 2 the LU traffic is medium. Finally, in Scenario 3 the LU traffic is mostly heavy.

In Figure 3, the average throughput, E$[\phi_s]$, achieved by each user connected to the small cell is shown as a function of the total number of MUs associated to the SAP, i.e. $M_1' + M_2'$. Remember that $M_2' = 0$ if $M_1' < M_1$. We consider the backhaul Scenario 1. Each curve corresponds to a different value of M_1, i.e. it corresponds to a different number of MUs in zone A_1. We can see that when M_1 is low, the highest throughput is achieved when the MUs are not allowed to connect to the SAP, while when the M_1 is high, the highest throughput is achieved when all the MUs in zone M_1 are

TABLE I
Transition rate matrices for different backhaul scenarios.

Scenario 1 Light			**Scenario 2** Medium			**Scenario 3** Heavy		
$\frac{-1}{900}$	$\frac{1}{360}$	$\frac{1}{540}$	$\frac{-1}{600}$	$\frac{1}{60}$	$\frac{1}{540}$	$\frac{-1}{480}$	$\frac{1}{60}$	$\frac{1}{420}$
$\frac{1}{18}$	$\frac{-1}{138}$	$\frac{1}{120}$	$\frac{1}{180}$	$\frac{-1}{540}$	$\frac{1}{360}$	$\frac{1}{240}$	$\frac{-1}{330}$	$\frac{1}{90}$
0	$\frac{1}{12}$	$\frac{-1}{12}$	0	$\frac{1}{12}$	$\frac{-1}{12}$	0	$\frac{1}{180}$	$\frac{-1}{180}$

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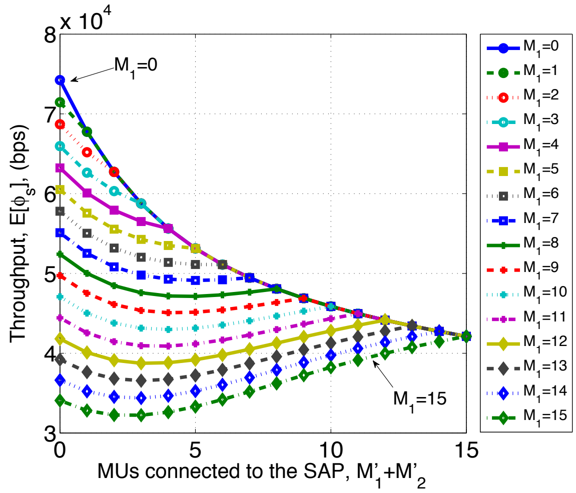

Fig. 3. Throughput achieved by users associated to the SAP as a function of the MUs associated to the SAP for different values of MUs in zone A_1.

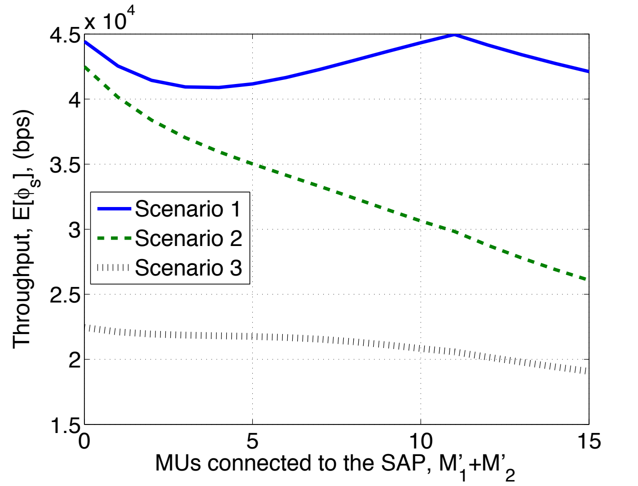

Fig. 4. Throughput achieved by users associated to the SAP as a function of the MUs associated to the SAP for different backhaul scenarios.

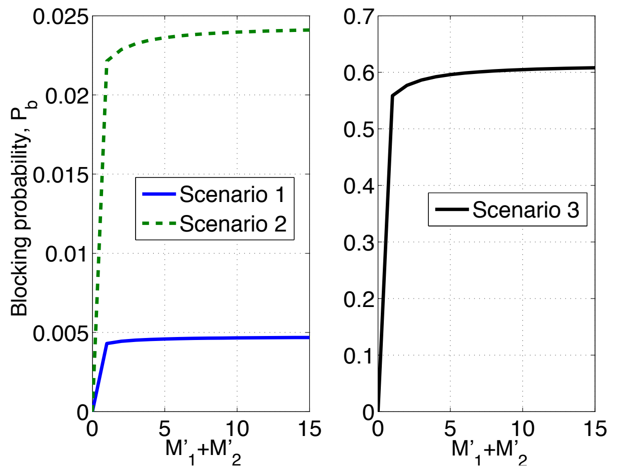

Fig. 5. Blocking probability as a function of the MUs associated to the SAP for different backhaul scenarios.

associated to the SAP. This can be explained as follows. When M_1 is low, the interference produced by the MUs is low and therefore the reduction of interference obtained when the MUs are associated to the SAP is not enough in order to compensate for the corresponding reduction of capacity. This is because the bandwidth is shared among all users connected to the SAP and therefore, when there are few users connected, connecting one more user entails a high reduction of the throughput. However, when M_1 is high, the interference produced by the MUs is high and connecting them to the SAP compensates for the reduction of capacity. Observing only one curve for a high M_1, we can see that the throughput first decreases, reaches a minimum and then increases until it reaches a maximum and decreases again. This can explained as follows. When there are few users connected, connecting one more user decreases the throughput of each user more than when there are many users connected. Therefore, when $M_1' + M_2'$ is low, the reduction of throughput due to the association of one more MU to the SAP is higher than the gain of throughput due to a reduction of interference. The opposite case occurs when $M_1' + M_2'$ increases and a maximum is achieved when all the MUs in zone A_1 are associated to the SAP. Then, if a user in zone A_2 is associated to the SAP, again the reduction of interference does not compensate for the reduction of bandwidth as the interference produced in zone A_2 is lower.

In Fig. 3, we can also see that for a given value of $M_1'+M_2'$ the value of E$[\phi_s]$ is the same for all the curves where $M_1 \leq M_1' + M_2'$, i.e., all the MUs in A_1 are associated to the FAP. This occurs as when $M_1 \leq M_1'+M_2'$, the total number of MUs associated to the FAP is $M_1'+M_2'$, the number of MUs in A_1 associated to the BS is 0, the number of MUs in A_2 associated to the BS is $M_2 - M_2' = (15 - M_1) - (M_1' + M_2' - M_1) = 15 - (M_1' + M_2')$; and none of these numbers depends on M_1.

In Fig. 4, the average throughput E$[\phi_s]$ is shown again as a function of $M_1' + M_2'$. We consider $M_1 = 11$. Each curve corresponds to one of the three different backhaul scenarios shown in Table I. As it can be expected, for the scenarios where the LU traffic is higher, the throughput achieved is lower. We can see that for Scenario 1, where the LU traffic is mostly light, the highest throughput is achieved when all the MU in zone A_1 are associated to the SAP. In Scenario 2, where the LU traffic is medium, we can see that the backhaul is more restrictive than the interference produced by the MUs connected to the BS. Therefore, the association of more MUs to the SAP entails a reduction of throughput. For Scenario 3, where the LU traffic is mostly heavy, the average throughput declines as $M_1' + M_2'$ increases, but at much slower rate than in Scenario 2. This is because the blocking probabilities (see Fig. 5) are very high, and the system is serving the users with a bitrate closer to their minimum. Thus, in this figure, we can see the importance of the backhaul state when deciding how many MUs should be associated to the SAP.

In Fig. 5, the blocking probability of the users associated to the SAP is shown as a function of $M_1' + M_2'$. We consider

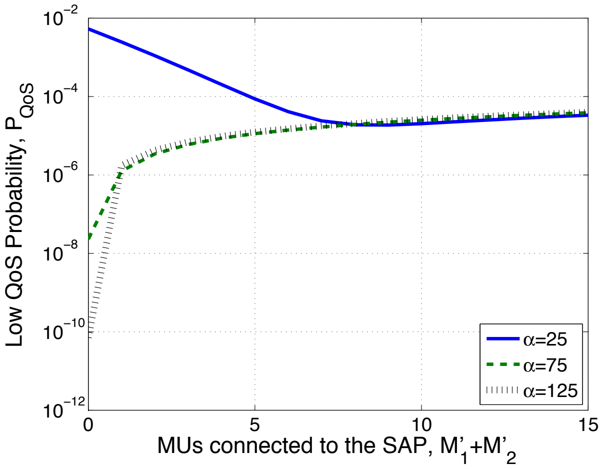

Fig. 6. Low QoS probability as a function of the MUs associated to the SAP for different values of α.

$M_1 = 11$. Each curve corresponds to one of the three different backhaul scenarios shown in Table I. We can see that as expected, the blocking probability is higher when more MUs are associated to the SAP. The blocking probability is 0 when the MUs are not allowed to connect to the SAP since the minimum bitrate available to the SAP by the backhaul guarantees that all the SUs are served at their minimum bitrate. For $M_1' > 0$, as expected, the blocking probability is higher when the LU traffic is higher. For Scenario 3, the blocking probabilities are very high.

In Fig. 6, the low QoS probability of the users associated to the SAP is shown as a function of $M_1' + M_2'$. We consider the backhaul Scenario 1 and $M_1 = 11$. Each curve corresponds to a different α, i.e. a different duration of an idle period. For low values of α, i.e. less loaded systems, the low QoS probability first decreases, then reaches a minimum and increases again at a slow rate. This is because for low values of $M_1' + M_2'$, the interference produced by the MUs associated to the BS has more influence on the low QoS probability than the backhaul state. When $M_1' + M_2'$ is high, it is the opposite case, the backhaul state has more influence than the interference. However, for scenarios with higher α, the minimum disappears because the system is more loaded and the backhaul state has more influence than the interference. A sharp increase for low values of $M_1' + M_2'$ appears. This occurs as when $M_1' + M_2' = 0$, the backhaul guarantees the minimum bitrate for all the users connected to the SAP and the low QoS is produced only by the interference from the MUs. When one MUs is associated to the SAP, the low QoS is produced by the backhaul. Moreover, the curve for $\alpha = 125$ has a start sharper than the curve for $\alpha = 75$. This is because for $M_1' + M_2' = 0$ the low QoS probability is lower for higher α as for high α the system is longer in the blocking states, and shorter in states with low QoS. Finally, note that when the low QoS is produced by the backhaul the curves are very close as the backhaul scenario is the same for the three curves.

V. Conclusions

In this paper, we present an analytical model to study the impact of the traffic dynamics and backhaul constraints on the small cell architecture. We use this model to evaluate the small cell performance depending on the number of MUs allowed to connect to the small cell. We model the interference by considering different geographical regions of interference. We model the load generated by the land-line users by using finite-state CTMCs. Then, the system is modeled by using a multidimensional CTMC.

The results show that the AC policy which maximizes the average throughput achieved by the SUs depends first on the backhaul state. When the backhaul is highly loaded, a closed access should be implemented in order to maximize the throughput and minimize the blocking probability of the SUs. When the backhaul is not highly loaded, the AC policy depends on the interference produced by the MUs. When the interference is low, the closed access mode is better. When the interference is high, the MUs in the region closer to the SAP should be associated to the SAP. Moreover, we found that the AC policy that minimizes the low QoS probability depends on the load of the small cell.

For future works, dynamics at longer time scales than the flow dynamics can be considered and thus, the model can include a variable number of users. Moreover, this model can be further used to analyze more complex behaviors of the backhaul and also to evaluate other performance parameters such as the average time that the QoS is lower than the required.

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