Insensitive Call Admission Control for Wireless Multiservice Networks

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***Abstract*—We propose a call admission control policy and prove that the CTMC that models the multiservice wireless system enforcing the CAC scheme is reversible and its stationary distribution is insensitive to the channel holding time distribution.**

***Index Terms*—Call admission control, call blocking probability, quality of service, wireless networks.**

I. INTRODUCTION

TWO important quality of service measures for wireless networks are the fraction of new and handover calls that are blocked due to the lack of enough free resources. As handover blocking is more annoying than new call blocking for subscribers, efficient call admission control (CAC) strategies can be used to reject new calls in order to reserve resources for future handovers, while minimizing the impact on the blocking rate of new calls.

Conventional trunk reservation policies for CAC lead to continuous-time Markov chains (CTMC) whose state-space cardinality grows very quickly with the number of channels and services supported. Then, determining the stationary distribution and parameters derived from it, like new and handover probabilities, might become an unfeasible task [1].

We propose a probabilistic CAC policy for multiservice wireless networks that supports different service classes (SCs) and provides differentiated treatment to each arrival type (new or handover). The CTMC that models the system is reversible and its stationary distribution has a product-form, which greatly simplifies its computation. In addition, the stationary distribution is insensitive to the channel holding time (CHT) distribution. An interesting feature of the proposed policy is that the resource sharing among SCs, and between new and handover calls of the same SC, can be controlled independently. Our work has been motivated in part by the study presented in [2], although we obtain results different from the ones derived there.

In the next section we prove the reversibility and insensitivity properties. For illustrative purposes, in Section III we present a numerical example that confirms the insensitivity property. Finally, in Section IV we present examples of both reversible and non-reversible CAC policies.

This work was supported by the Spanish Government through projects TIN2008-06739-C04-02 and TIN2010-21378-C02-02.

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II. REVERSIBILITY AND INSENSITIVITY

Consider a cellular network with C channels that supports J SCs. Since new and handover requests are distinguished, the system handles $2J$ arrival types. We assume that new and handover arrivals of the jth SC occur according to a Poisson process with rates λ_{nj} and λ_{hj}, respectively. The CHT in a cell is the minimum of the call duration and the cell residence (dwell) time. We assume that it is exponentially distributed. As shown below, this assumption on the CHT has no impact on the results. We denote by μ_{nj} and μ_{hj} the CHT rates for new and handover arrivals of the jth SC.

Let n_{nj} and n_{hj} be the number of ongoing calls of the jth SC, $1 \leq j \leq J$, initiated as new or handover requests, respectively. For arrivals of the jth SC, we define the following admission policy at state $\boldsymbol{n} = (n_{n1}, \ldots, n_{nJ}, n_{h1}, \ldots, n_{hJ})$: i) a *new* arrival is accepted with probability $a_j(n_{nj})\, b_j(n_{nj} + n_{hj})$; ii) a *handover* arrival is accepted with probability $b_j(n_{nj} + n_{hj})$; where

$$\begin{aligned} \boldsymbol{a_j} &= [a_j(0), a_j(1), \ldots, a_j(M_j - 1), 0] \ , \\ \boldsymbol{b_j} &= [b_j(0), b_j(1), \ldots, b_j(M_j - 1), 0] \ , \end{aligned} \tag{1}$$

$0 \leq a_j(m), b_j(m) \leq 1$; $0 \leq m \leq (M_j - 1)$; $M_j = \lfloor C/c_j \rfloor$; and c_j is the number of channels required to set up a call of the jth SC. Clearly, $\boldsymbol{a_j}$ and $\boldsymbol{b_j}$ are vectors of probabilities, and M_j is the maximum number of ongoing calls of the jth SC (either initiated as new or handover) in the system.

Let us denote by $c(\boldsymbol{n}) = \sum_{j=1}^{J} (n_{nj} + n_{hj})\, c_j$ the number of channels occupied in state $\boldsymbol{n}$. Then, the CTMC $\{\boldsymbol{n}(t)\}_{t \geq 0}$ with state space $\mathcal{S} := \{\boldsymbol{n} \mid c(\boldsymbol{n}) \leq C, n_{nj}, n_{hj} \in \mathbb{N}\}$ is reversible. For convenience, we prove it by showing that the so called arrival and service processes of an equivalent queuing network are reversible [3].

Consider a queuing network with $2J$ nodes, no waiting facilities (loss network) and no internal routing, where new arrivals of the jth SC are offered to node j and handover arrivals of the jth SC are offered to node $j + J$. Assume Poisson arrivals from outside the network with rates $\lambda_j = \lambda_{nj}$ and $\lambda_{j+J} = \lambda_{hj}$, and exponentially distributed services with rates $\mu_j = \mu_{nj}$ and $\mu_{j+J} = \mu_{hj}$, $1 \leq j \leq J$. Let $\boldsymbol{x} = (x_1, \ldots, x_{2J})$ be the vector whose ith component gives the number of ongoing calls at node i, $1 \leq i \leq 2J$. In state $\boldsymbol{x}$, an arrival to node j is accepted with probability $a_j(x_j)\, b_j(x_j + x_{j+J})$, while an arrival to node $j + J$ is accepted with with probability $b_j(x_j + x_{j+J})$. In addition, admission decisions are subject to the capacity constraint $c(\boldsymbol{x}) \leq C$.

Let us define the following transition rates for the CTMC $\{\boldsymbol{x}(t)\}_{t\geq 0}$: i) $q(\boldsymbol{x}, \boldsymbol{x}+\boldsymbol{e_i}) = \gamma_i(\boldsymbol{x})$, if $c(\boldsymbol{x}+\boldsymbol{e_i}) \leq C$; ii) $q(\boldsymbol{x}, \boldsymbol{x}-\boldsymbol{e_i}) = \mu_i(\boldsymbol{x})\, p_i(\boldsymbol{x}) = \mu_i(\boldsymbol{x})$; and iii) $q(\boldsymbol{x}, \boldsymbol{x}-\boldsymbol{e_i}+\boldsymbol{e_k}) = \mu_i(\boldsymbol{x})\, p_{ik}(\boldsymbol{x}) = 0$, where $\boldsymbol{e_i}$ is a $2J$-dimensional vector with component i set to 1 and 0 elsewhere, $\mu_i(\boldsymbol{x}) = x_i \mu_i$ and

$$\gamma_i(\boldsymbol{x}) = \begin{cases} a_i(x_i)\, b_i(x_i + x_{i+J})\, \lambda_i & \text{if } 1 \leq i \leq J\ , \\ b_{i-J}(x_{i-J} + x_i)\, \lambda_i & \text{if } J+1 \leq i \leq 2J\ . \end{cases}$$

We consider that after service completion at node i in state $\boldsymbol{x}$, a call is routed to node k with probability $p_{ik}(\boldsymbol{x}) = 0$, and leaves the network with probability $p_i(\boldsymbol{x}) = 1$. Additionally, $\gamma_i(\boldsymbol{x})$ is the effective arrival rate to node i in state $\boldsymbol{x}$, which takes into account the impact of the admission policy. Then, the CTMC $\{\boldsymbol{x}(t)\}_{t\geq 0}$ that describes the dynamics of the queuing network is the same as $\{\boldsymbol{n}(t)\}_{t\geq 0}$, the one that describes the multiservice wireless system under study.

For the considered queuing network, if there is a positive function Φ that satisfies $\forall i$, $1 \leq i \leq 2J$, and $\forall \boldsymbol{x} \in \mathcal{S}$ that

$$\Phi(\boldsymbol{x}) = \Phi(\boldsymbol{x}+\boldsymbol{e_i})\, \mu_i(\boldsymbol{x}+\boldsymbol{e_i})\ , \tag{2}$$

then the service process is reversible [3]. The function $\Phi(\boldsymbol{x}) = \prod_{i=1}^{2J} 1/\left(x_i!\ \mu_i^{x_i}\right)$ meets condition (2).

Likewise, if there is a positive function Λ that satisfies $\forall i$, $1 \leq i \leq 2J$, and $\forall \boldsymbol{x} \in \mathcal{S}$ that

$$\Lambda(\boldsymbol{x})\, \gamma_i(\boldsymbol{x}) = \Lambda(\boldsymbol{x}+\boldsymbol{e_i})\ , \tag{3}$$

then the arrival process is reversible [3]. Condition (3) is met by function $\Lambda(\boldsymbol{x}) = \prod_{i=1}^{2J} \lambda_i^{x_i} \prod_{j=1}^{J} \alpha_j(x_j)\, \beta_j(x_j + x_{j+J})$, where $\alpha_j(u) = \prod_{k=0}^{u-1} a_j(k)$ and $\beta_j(u) = \prod_{k=0}^{u-1} b_j(k)$.

Thus, the stationary distribution of the CTMC that describes the dynamics of the considered queuing network becomes $P(\boldsymbol{x}) = P(\boldsymbol{0})\, \Lambda(\boldsymbol{x})\, \Phi(\boldsymbol{x})$, $\boldsymbol{x} \in \mathcal{S}\backslash\{\boldsymbol{0}\}$, where $P(\boldsymbol{0})$ is obtained by normalization [3]. Equivalently, we obtain

$$P(\boldsymbol{n}) = P(\boldsymbol{0}) \prod_{j=1}^{J} \prod_{r=0}^{n_j-1} b_j(r) \prod_{s=0}^{n_{nj}-1} a_j(s) \frac{\rho_{nj}^{n_{nj}}}{n_{nj}!} \frac{\rho_{hj}^{n_{hj}}}{n_{hj}!}\ , \tag{4}$$

where $n_j = n_{nj} + n_{hj}$, $\rho_{nj} = \lambda_{nj}/\mu_{nj}$ and $\rho_{hj} = \lambda_{hj}/\mu_{hj}$ [1]. Then, the blocking probabilities can be determined by

$$P_{nj}^B = 1 - \sum_{n \in \mathcal{S}} a_j(n_{nj})\, b_j(n_{nj} + n_{hj})\, P(\boldsymbol{n})\ ,$$

$$P_{hj}^B = 1 - \sum_{n \in \mathcal{S}} b_j(n_{nj} + n_{hj})\, P(\boldsymbol{n})\ ,$$

where $a_j(M_j) = b_j(M_j) = 0$ as defined in (1).

When both the arrival and service processes are reversible, then the queuing network process $\{\boldsymbol{x}(t)\}_{t\geq 0}$, and therefore $\{\boldsymbol{n}(t)\}_{t\geq 0}$, are also reversible. In addition, their stationary distributions are *insensitive*, in the sense that they depend on the service time distribution at each node through the mean only. In other words, when arrivals follow Poisson processes, all key performance indicators obtained from the stationary distribution, like blocking probabilities, are independent from all traffic characteristics beyond the traffic intensity [3].

[1]Expression (4) leads to results different from the ones derived in [2]. For a single SC system, setting $J = 1$ in (4) does not yield expression (3) of [2]. Also, expression (11) of [2] is not consistent with (4).

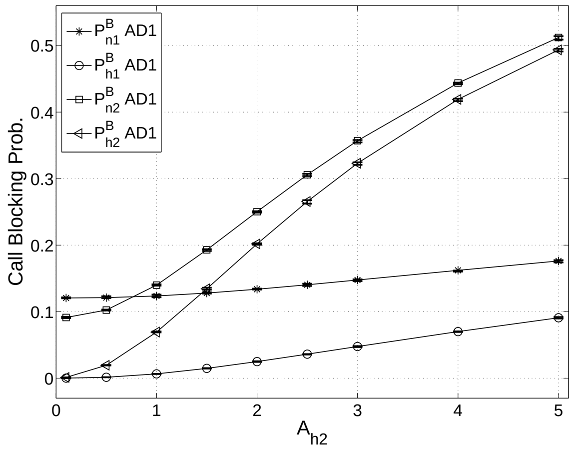

Fig. 1. Service times are hyperexponentialy distributed with $CV = 4$.

III. Numerical Evaluation

For illustrative purposes, we compare the blocking probabilities of the different arrival types obtained by equation (4) with those obtained by simulation when service distributions (CHTs) other than the exponential distribution are used, like Erlang, hyperexponential, lognormal and bounded Pareto. The results confirm the insensitivity property and the correctness of (4).

Let the admission probabilities be defined by

$$a_j(k) = \begin{cases} A_j^d & \text{if } 0 \leq kc_j < K_j\ , \\ A_j^u & \text{if } K_j \leq kc_j < M_j c_j\ , \end{cases}$$

$$b_j(k) = \begin{cases} B_j^d & \text{if } 0 \leq kc_j < C_j\ , \\ B_j^u & \text{if } C_j \leq kc_j < M_j c_j\ . \end{cases}$$

This policy is a subclass of the one defined in Section II, and therefore all previous results apply. Note that when $A_j^d = B_j^d = 1$ and $A_j^u = B_j^u = 0$, the resource sharing between the SCs can be controlled by configuring $\{C_j\}$, while the resource sharing between arrival types of the same SCs by configuring $\{K_j\}$.

The system we study is defined by: $J = 2$, $C = 30$, $c_1 = 1$, $c_2 = 6$, $\lambda_{n1} = 1/20$, $\lambda_{n2} = 1/50$, $\lambda_{h1} = 1/25$, $\lambda_{h2} = 1/55$, $\mu_{n1} = 1/100$, $\mu_{n2} = 1/5$, $\mu_{h1} = 1/100$ and μ_{h2} is chosen to achieve that the traffic offered by handover arrivals of the SC 2 is within the interval $0.1 \leq A_{h2} = \lambda_{h2}/\mu_{h2} \leq 5.0$. The admission policy is defined as above with $K_1 = 7$, $K_2 = 6$, $C_1 = 20$ and $C_2 = 30$. Then, a new arrival of jth SC in state $\boldsymbol{n}$ is accepted with probability $A_j^d B_j^d = 1$, if $n_{nj} c_j < K_j$ and $(n_{nj} + n_{hj})\, c_j < C_j$, and rejected otherwise. While a handover arrival is accepted with probability $B_j^d = 1$, if $(n_{nj} + n_{hj})\, c_j < C_j$, and rejected otherwise.

In Fig. 1 continuous line curves were obtained using the distribution in (4). Simulation results overlap the analytical ones and therefore we only drew the 99% confidence intervals, which are very narrow and almost inperceptible. Note that CV refers to the coefficient of variation. Although not shown here due to lack of space, narrower confidence intervals were obtained for the Erlang ($CV = 0.25$), lognormal ($CV = 1.0$) and bounded Pareto distributions. More precisely, for blocking

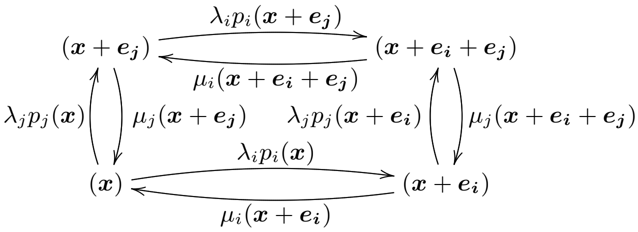

Fig. 2. State and transition diagram loop.

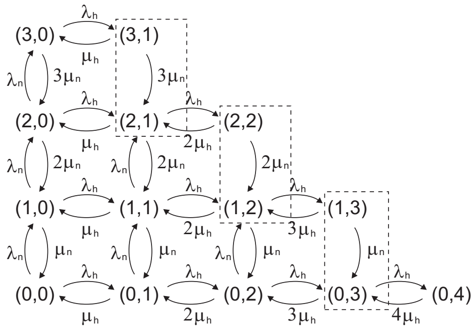

Fig. 3. State and transition diagram of a single service system.

probabilities above 1% the relative error defined as the radius of the confidence interval divided by the blocking probability value is lower than 3% for the last three distributions, while it is lower than 5.5% for the hyperexponential one (shown in Fig. 1). For the bounded Pareto distribution we used the definition in [4]. We set the shape factor to $\alpha = 2.001$, the maximal value to $H = 10^5$ and adjusted the minimal value (L) accordingly to achieve the desired mean, obtaining CVs in the interval $[1.51, 2.33]$.

IV. Discussion

The CTMC that models the queuing network defined in Section II is reversible if the Kolgomorov criterion is met for all possible loops of the transition diagram [5], [6]. From the loop shown in Fig. 2, the following condition is obtained:

$$\frac{p_i\left(\boldsymbol{x}+\boldsymbol{e_j}\right)}{p_i\left(\boldsymbol{x}\right)} = \frac{p_j\left(\boldsymbol{x}+\boldsymbol{e_i}\right)}{p_j\left(\boldsymbol{x}\right)} , \tag{5}$$

where $p_i\left(\boldsymbol{x}\right)$, $1 \leq i \leq 2J$, is the probability that an arrival to the ith node in state $\boldsymbol{x}$ is accepted.

The well known multiple Guard Channel policies in mobile networks are threshold type trunk reservation policies, also known as *cutoff priority* policies. For them, $p_i\left(\boldsymbol{x}\right) = I\left(c\left(\boldsymbol{x}+\boldsymbol{e_i}\right) \leq t_i\right)$, where I is an indicator function that is 1 when the condition is met and 0 otherwise. Then, type-i arrivals see a system limited to t_i channels and are accepted depending on the occupation at arrival time. It is not difficult to show that $\forall i, k, 1 \leq i, k \leq 2J$, condition (5) requires that $t_i = t_k$, i.e., all SCs share the same threshold. As a consequence, no differentiated treatment can be provided, neither among SCs, nor between new and handover arrivals of the same SC. In fact, the policy degenerates into a *Complete Sharing* policy.

Conversely, if a trunk reservation policy was used, full bidirectional connectivity between adjacent states of the CTMC might be lost and therefore the detailed balance equations would not hold. As detailed balance is a necessary condition for reversibility [6], then the CTMC would not be reversible. For illustrative purposes, Fig. 3 shows the state diagram of the CTMC modeling a one cell system enforcing a trunk reservation policy. The system parameters are: $J = 1$, $C = 4$ and $c_1 = 1$. Then, a new arrival is accepted with probability $p\left(n_{n1} + n_{h1}\right) = 1$, if $\left(n_{n1} + n_{h1}\right) < 3$, and rejected otherwise. While handover arrivals are always accepted if free resources are available. Note that bidirectional connectivity is lost for the adjacent states inside the discontinuous line boxes.

While trunk reservation policies are classical policies that take into account the total system occupation, if $\{p_i\left(\boldsymbol{x}\right)\}$ are a function of the total number of active calls, i.e. $\{p_i\left(b\left(\boldsymbol{x}\right)\right)\}$, $b\left(\boldsymbol{x}\right) = \sum_{j=1}^{J}\left(x_j + x_{j+J}\right)$, then a new family of reversible policies can be obtained. Let us define $\delta\left(m\right) = p_i\left(m\right)/p_i\left(m-1\right)$ and $\varphi_i = p_i\left(0\right)$. Then, $\gamma_i\left(\boldsymbol{x}\right) = \lambda_i p_i\left(M\right) = \lambda_i\varphi_i\prod_{m=1}^{M}\delta\left(m\right)$ is the arrival rate to the ith node in state $\boldsymbol{x}$, where $M = b\left(\boldsymbol{x}\right)$. Functions $\Phi\left(\boldsymbol{x}\right) = \prod_{i=1}^{2J} 1/\left(x_i!\,\mu_i^{x_i}\right)$ and $\Lambda\left(\boldsymbol{x}\right) = \prod_{i=1}^{2J}\left(\lambda_i\varphi_i\right)^{x_i}\prod_{m=1}^{M}\delta\left(m\right)^{M-m}$, meet conditions (2) and (3), respectively. Therefore, the CTMC that models the queuing network being considered, and therefore the associated multiservice wireless network, is reversible and its stationary distribution

$$P\left(\boldsymbol{x}\right) = P\left(\boldsymbol{0}\right)\prod_{i=1}^{2J}\frac{\left(\rho_i\varphi_i\right)^{x_i}}{x_i!}\prod_{m=1}^{M}\delta\left(m\right)^{M-m} ,$$

where $\rho_i = \lambda_i/\mu_i$, is insensitive to the CHT distribution.

Thus, in contrast to what is suggested in [2], trunk reservation policies do not lead to reversible CTMCs unless further restrictions are imposed. As an example, the *Thinning Scheme I* proposed in [7] requires that $\forall j, k$, $1 \leq j, k \leq J$, $c_j = 1$ and $\mu_{nj} = \mu_{hj} = \mu_{nk} = \mu_{hk}$. These conditions make the multidimensional CTMC to degenerate into a one dimensional birth and death process, which is known to be reversible and for which a product-form solution exists. The scheme includes the guard channel and the fractional guard channel schemes as special cases, although this only applies within the restricted scenario defined above.

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