Estimating the number of contending users for a single random access in LTE-A networks: The baseline for designing congestion control schemes at the evolved Node B *

Luis Tello-Oquendo, Israel Leyva-Mayorga, Vicent Pla, Jorge Martinez-Bauset
and Vicente Casares-Giner

ITACA. Universitat Politècnica de València. Camino de Vera s/n.
46022 Valencia, Spain

Abstract. Machine-type communications (MTC) have the potential to generate a myriad of connection requests in a short period. Using cellular networks to provide MTC connectivity presents numerous advantages such as coverage, roaming support, well-developed charging, QoS, security solutions, among others. Nevertheless, critical problems like congestion and overload of radio access and core networks need to be addressed for efficient cellular MTC. In LTE-A, the physical random access channel (PRACH) is used by MTC devices (UEs) to access the network. For doing so, a UE randomly chooses a preamble from a pool of preambles and transmits it during the PRACH. The evolved Node B (eNB) acknowledges the successful reception of a preamble if only one UE transmits that preamble. To increase the success rate of a massive number of access attempts is necessary to design congestion control schemes. For that, the key piece of information is the total number of UEs competing in the PRACH. To estimate this number at the eNB, we find the joint probability distribution function (PDF) of the number of successful and collided preambles within a random access slot. Then, we design a maximum likelihood estimator using this PDF. To further improve our estimation, we propose an iterative approach. Numerical results showcase the accuracy and usefulness of the proposed method even if the number of access attempts is significant.

Keywords: LTE-A networks, machine-type communications (MTC), maximum likelihood estimation, random access, random access channel (RACH) procedure.

1 Introduction

Nowadays, the use of cellular network technologies such as Long Term Evolution Advanced (LTE-A) for providing machine-type communications (MTC) has

attracted significant attention from both the research community and the industry. The widely deployed infrastructure is a major driving force that motivates MTC application developers to adopt cellular networks for their numerous remote monitoring and controlling applications [7, 16, 19]. However, critical problems like congestion and overload of radio access and core networks need to be addressed for efficient cellular MTC [15, 25].

In LTE-A, a random access procedure is initiated when a MTC device (named UE herein) desires to access the cellular network. The Physical Random Access Channel (PRACH) is used to signal a connection request. For that, the evolved Node B (eNB) has multiple preambles in a preamble pool available for initial access to the network; these preambles are generated by Zadoff-Chu (ZC) sequences due to their good correlation properties [4, 14]. The random access procedure consists of a four-message handshake. In *Msg1*, a UE transmits a randomly chosen preamble from the preamble pool during one of the available random access slots. A preamble will be detected at the eNB if it is not chosen by more than one UE in the same random access slot. Then, the eNB sends a random access response (RAR) message, *Msg2*, which includes one uplink grant for each detected preamble. *Msg2* is used to assign time-frequency resources to the devices for the transmission of *Msg3*. Next, the UEs that received an uplink grant send their connection request message, *Msg3*, during the resources specified by the eNB. Finally, the eNB responds to each *Msg3* transmission with a contention resolution message, *Msg4*. The interested reader is referred to [1, 3, 5, 6, 11, 12, 17, 20] for further details.

The contention-based random access procedure detailed before is similar to the slotted ALOHA protocol [10]; after the transmission of *Msg1*, a UE can be connected if there is no collision. In this sense, the random access procedure can be seen as a multichannel ALOHA [9], where congestion control is studied by estimating the number of arrivals or the number of UEs that send preambles to the eNB [13].

The estimation of the network load is a challenging task when the network includes MTC, due to the high (and unpredictable) number of UEs expected to access the cellular network simultaneously. In recent research, approaches using non-trivial combinatorics have been developed to derive the probability distribution of the number of successful and collided users in one-shot random access [21, 22]. However, the computational complexity of these procedures is considerably high and may not be suitable in the presence of thousands of UEs.

In this paper, we propose a simple scheme to estimate the network load. First, we design a recursive approach to find the joint probability distribution function (PDF) of successful, collided, and not used preambles in a random access slot as the number of UEs increases in the system. Then, based on this information available at the eNB, we find the number of contending UEs in a random access slot employing the maximum likelihood estimation to reduce the complexity during its on-line operation. Finally, we use an iterative approach to improve further the estimation.

The remainder of the paper is organized as follows. In Section 2, the system model and the problem formulation are presented. In Section 3, the method for estimating the number of contending users at eNB and the iterative approach for

refining its accuracy are detailed. Numerical results are presented in Section 4. Finally, the conclusions and future work are presented in Section 5.

2 System Model

This paper considers a fixed number of UEs performing random access to an LTE-A eNB. It is assumed that all the UEs fall within the coverage of just one eNB. According to the LTE-A standard [3,5,6], when a UE intends to establish a connection to an eNB, the UE initiates a random access procedure by sending a preamble to the eNB, *Msg1*, via a time-frequency radio resource called physical random access channel (PRACH). There are up to 64 orthogonal preambles available to the UEs per cell. Preambles are split into two sets:

– Contention-free: it is used for critical situations such as handover, downlink data arrival or positioning, where there is a coordinated assignment of preambles so collision is avoided. The eNB can only assign these preambles for specific slots to specific UEs. They can only use them, if assigned by the eNB, and for the specific slots assigned.
– Contention-based: it is the standard mode for network access (there are more preambles in this set). Preambles are selected in a random fashion, so there is risk of collision, i.e., multiple UEs in the cell might pick the same preamble signature and the eNB would assign the same physical resources to both UEs; therefore contention resolution is needed.

Herein, we focus on the *Msg1* of the contention-based random access procedure, and assume that UEs that have transmitted successfully the *Msg1* will successfully complete the random access procedure [8, 23]. The time is divided into independent fixed-length random access slots (RAS). Regarding the channel resources, the eNB has a set of r preambles $\{1 \leq r \leq 54\}$ available for contending UEs in each RAS, the remaining 10 out of 64 preambles are used in the contention-free mode by the eNB. Further, we assume that the cell is large enough, which allows the eNB to differentiate the preambles chosen by more than one UE, i.e. collisions in the preamble space can be detected [8, 25].

As explained before, if a device wants to connect to the eNB, it sends a preamble, chosen in a random fashion among all r preambles, in the first coming RAS. If the eNB receives this preamble without collision, the connection from the UE to the eNB can be established. However, if multiple UEs have transmitted the same preamble, a collision occurs, and the eNB ignores the collided preamble. A device learns the success or failure of its random-access attempt immediately at the end of the RAS [13, 22].

2.1 Capacity of the PRACH

In [18], it is found that the capacity of the PRACH, L, defined as the maximum expected number of preambles transmitted by a single UE in a RAS, approximately corresponds to the maximum number of stationary UE arrivals per RAS that the PRACH can handle efficiently, $\hat{L}$. In other words, the performance of the PRACH drops whenever the number of UEs (N)

that begin its RA procedure at the each and every RAS is $N \geq \hat{L}$. If r is the number of available preambles and N_i is the number of preamble transmissions at the ith RAS, the expected number of preambles transmitted by a single UE is $N_i (1 - 1/r)^{N_i - 1}$ and its maximum, L, is achieved when $N_i = [\log(r/[r-1])]^{-1} \approx r$, given as follows

$$L = \left[\log\left(\frac{r}{r-1} \right) \right]^{-1} \left(1 - \frac{1}{r} \right)^{\left[\log\left(\frac{r}{r-1} \right) \right]^{-1} - 1}, \tag{1}$$

which results, for instance, in $L = 20.05$ preambles transmitted by a single UE at a given RAS when $r = 54$ (see Fig. 1(a)). Furthermore, we observed that (1) can be approximated as follows

$$L \approx r \left(1 - \frac{1}{r} \right)^{r-1} \approx \frac{r}{e}. \tag{2}$$

Hence, assuming a typical PRACH configuration (*PRACH configuration index 6*, in conformance to the LTE-A specification [2,3]), the PRACH can handle a maximum of $\hat{L} \approx 20.05$ stationary UE arrivals per RAS and, given that RA slots occur every $T_{\mathrm{RAS}} = 5\,\mathrm{ms}$, a maximum of $\hat{L}/T_{\mathrm{RAS}} = 4010$ stationary UE arrivals per second.

3 Estimating the Number of Contending UEs at eNB

Some congestion control mechanisms, such as access class barring and extended access barring in LTE-A [3], require complete awareness of the input load to optimize their parameters or adapt them to the network status. The input load depends on the arrival process of the access requests and can only be estimated from the information available at the eNB during a time slot, i.e., the number of success/collided/not-used preambles in an RAS [12, 24].

Let us focus on a single RAS with r preambles, and, to simplify the notation, let the random variable N be the number of UEs which randomly select their transmission preamble among the r preambles available in the RAS. Denote by S the random variable representing the number of preambles successfully transmitted, C the random variable representing the number of collided preambles and U the random variable representing the number of not-used preambles. The conditional joint probability that models the fact that exactly $S = s$ and $C = c$ out of $N = n$ UEs transmit their preambles successfully and with collision, respectively, becomes

$$P_n(s, c) = P(S = s, C = c, U = r - s - c | N = n) = \frac{\left(\begin{smallmatrix} & r & \\ u & s & c \end{smallmatrix} \right) \left\langle \begin{smallmatrix} c \\ n-s-2c \end{smallmatrix} \right\rangle}{\left\langle \begin{smallmatrix} n \\ r \end{smallmatrix} \right\rangle}; \tag{3}$$

therefore, the conditional probability that exactly s out of the n UEs successfully transmit their preamble in a RAS comprising r preambles is given by

$$P_n(s) = P(S = s | N = n) = \sum_c P_n(s, c). \tag{4}$$

(a) Maximum expected number of preambles transmitted by a single UE per RAS given the number of available preambles, r.

(b) Diagram of the recursive approach for computing $P_n(s, c)$ according to a chosen preamble.
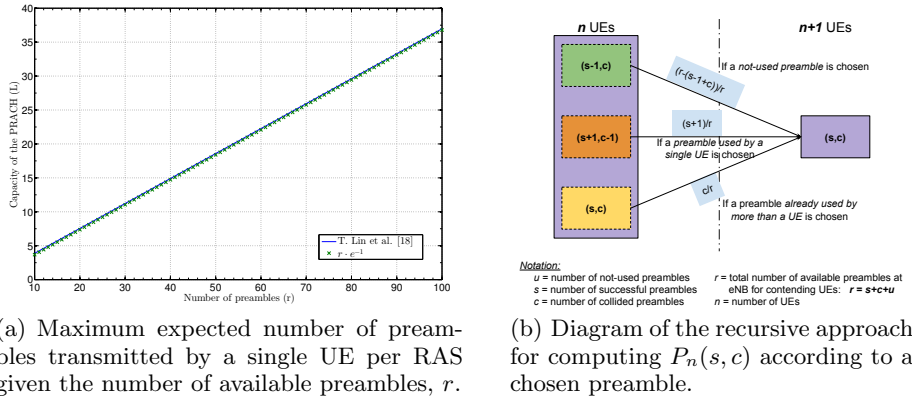
Fig. 1. System capacity and recursive approach diagram for computing $P_n(s, c)$ as the number of UEs, n, increases in the system.

As MTC imply a massive number of UEs accessing the network, it is necessary to find an efficient way to compute the conditional joint probability distribution, $P_n(s, c)$, for a great number of UEs registered in the system. For doing so, in Fig. 1(b) we illustrate what happens at the eNB when a UE performs a new access attempt in a determined RAS; the diagram represents the number of s and c preambles (s, c) detected in the RAS when there are a certain number of users, n, attempting access at the eNB. If the number of UEs increases in the RAS, $n + 1$, a new preamble must be selected so that we can identify three outcomes:

1. There are $(s - 1, c)$ preambles detected at the eNB, and the arriving UE has chosen one of the not-used preambles; in that case, it is a successful access attempt, and the detected preambles becomes (s, c).
2. There are $(s + 1, c - 1)$ preambles detected at the eNB and the arriving UE has chosen a preamble used by a single UE; in that case, a collision is produced, and the detected preambles becomes (s, c).
3. There are (s, c) preambles detected at the eNB, and the arriving UE selects a preamble already used by more than one UE; in that case, the eNB does not detect the preamble, remaining the same (s, c) preambles.

As a result of the described observations, we devised a recursive method for computing the conditional joint probability distribution function (PDF) for the number of successful and collided preambles in a RAS as the number of UEs in the system, n, increases. It is computed as follows

$$P_{n+1}(s, c) = \frac{r - (s - 1 + c)}{r} P_n(s - 1, c) + \frac{s + 1}{r} P_n(s + 1, c - 1) + \frac{c}{r} P_n(s, c). \quad (5)$$

Note that this operation is computationally tractable and this problem can be solved offline numerically for a great number of UEs (n). A look-up table is obtained with r rows r columns for different values of s and c, respectively.

Moreover, this table is computed once and can be used throughout the operation of the system.

Finally, we can estimate N by using the maximum likelihood estimation (MLE) from s as

$$\hat{N} \equiv \hat{N}(s) = \arg\max_n P_n(S = s). \tag{6}$$

3.1 Improving the initial estimation

As a baseline, the number of contending UEs in a RAS can be estimated using only the conditional probability that a device can transmit successfully its preamble by (6). Note that the number of the devices with successful preamble transmissions, S, is limited as $S \leq r$, since there are r preambles available in the RAS. Nevertheless, we can also use the number of collided preambles for the estimation of N. Specifically we can use the conditional joint PDF for the number of successful and collided preambles, $P_n(s, c)$, in the MLE as follows

$$\hat{N} \equiv \hat{N}(s, c) = \arg\max_n P_n(S = s, C = c). \tag{7}$$

In the simulations, as will be explained in the following Section 4, it can be observed that we get acceptable results estimating the number of contending UEs by either (6) or (7). Furthermore, we observe that $\hat{N}$ is always less than or equal to the real number of contending UEs. Therefore, based on the latter observation, the MLE can be improved through an iterative approach. For that, we consider that given an initial estimation of the contending UEs, $\hat{N}$, it can be refined as follows

$$\hat{N}^{(1)} = \frac{\hat{N}^2}{\mathbb{E}_n[\hat{N}]}, \tag{8}$$

where $\mathbb{E}_n[\hat{N}] = \sum_s \hat{N}(s)P_n(s)$ if the initial estimation, $\hat{N}$, has been computed by (6). On the other hand, if the initial estimation has been computed by (7), $\mathbb{E}_n[\hat{N}] = \sum_{s,c} \hat{N}(s, c)P_n(s, c)$. This refinement allows for an elegant solution that yields accurate results, as demonstrated further.

4 Numerical Results

In this section, we present the simulation results for getting the MLE of N from $P_n(s)$ and $P_n(s, c)$, respectively. We have performed 10000 runs for each value of N to find the mean values of the MLE. In real implementations, the number of UEs within a single cell could be significantly large, so we vary the number of UEs from 1 up to 100 times the capacity of the PRACH, L, computed using (1). For example, when $r = 54$, the PRACH capacity becomes $L = 20.05$; therefore, we vary the number of UEs from 1 up to 2005. We used different values for the number of preambles available in the system for the contention-based random access, $r \in \{1, 2, 3, \ldots, 54\}$. But we observed that the results are qualitatively similar for each value of r tested; therefore, next we show and analyze the ones corresponding to the most typical scenario (in conformance to
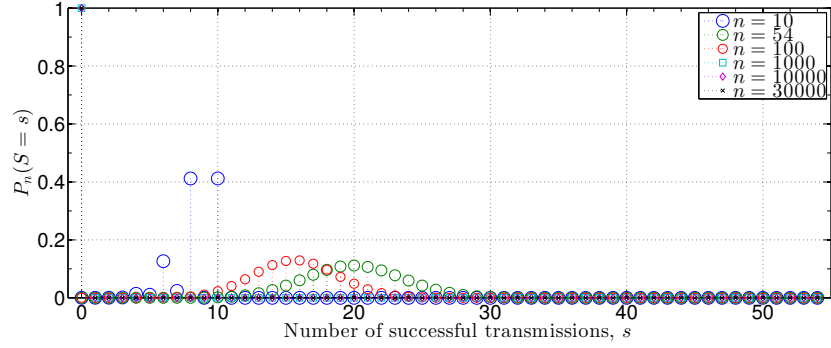
Fig. 2. Distribution of the number of successful transmissions in a given random access slot, $P_n(s)$, $r = 54$.
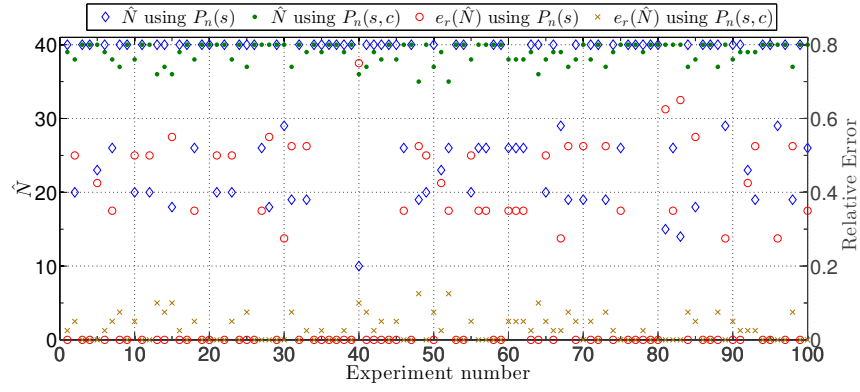

Fig. 3. MLE of contending UEs at eNB when $N = 40$, $r = 54$.

the LTE-A specification [2, 3]) when the eNB have $r = 54$ preambles available for the contention-based random access procedure.

As a baseline, Fig. 2 illustrates the probability distribution of the number of successful transmissions, $P_n(s)$, in a RAS comprising $r = 54$ preambles. As intuitively expected, the distribution is irregular when the number of UEs is low (see the curve of $n = 10$). As n increases, the curves makes a well-defined peak at a determined value of n; additionally, we see that the probability mass is concentrated around the mean value, as can be seen clearly in the curve of $n = 100$. In essence, Fig. 2 intuitively suggests that for a relatively large r, the PDF of the number of successful transmissions is highly concentrated around its mean value, for either n small and large [21].

Then, we analyze two cases for estimating the number of contending UEs at the eNB. First, we consider that the unique available information at the eNB in a time slot is the number of successful preamble transmissions, s, and find the MLE
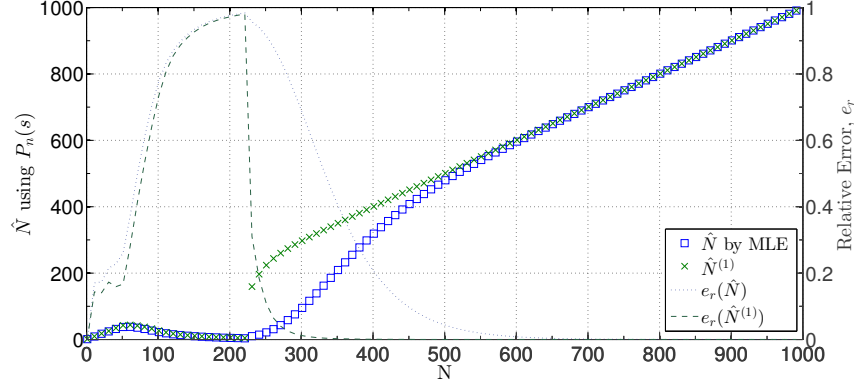
Fig. 4. MLE of contending UEs at eNB using $P_n(s)$, $r = 54$.

of N by (6). We observe that, for a test value $N = 40$ (see Fig. 3), the relative error of the estimated values ranges from 0 to 70% in the worst case. It might be a reasonable estimation; however, it can be improved considering additional information available at the eNB as is explained next. Second, we consider the fact that, at the eNB, the number of successful and collided preambles in a RAS is known. Thus, we can use the joint probability distribution (3) in the MLE of N. By doing so, equation (7) can provide much better results for any value of N, when compared to the ones obtained by (6). For example, see Fig. 3, for $N = 40$, the relative error in the estimations ranges from 0 to 14%.

One of the main observations in the results obtained by simulation, for instance the ones of Fig. 3, is that the error is upper-bounded by the real value of N, i.e., the estimation is always lower than or equal to the real number of contending UEs in a RAS. Taking advantage of this fact, an iterative approach, (8), can be devised. It allows us to increase further the accuracy of the estimation. Figures 4 and 5 illustrate the results following the iterative approach; we varied N from 1 up to 100 times the capacity of the PRACH computed using (1) as a function of the preambles available for contending UEs, $r = 54$.

On the one hand, in Fig. 4 we illustrate the estimation of the number of contending UEs by (6) from $P_n(s)$. Analyzing these results, we observe that in the extremes, $\{1 < N < 50\}$ and $\{N > 500\}$, the estimation is very accurate; whereas, there are values of N, $\{50 < N < 500\}$, in which the MLE is biased and the relative error of the estimation reaches the 98%. Clearly, an enhancement is necessary in these cases; therefore, we use the iterative approach by (8) for refining the initial estimation. This refinement yields a remarkable improvement in the accuracy at $\{N > 222\}$ and we observe that the enhancement especially improves the accuracy in cases where the initial estimation was longer acceptable. Conversely, when the estimation accuracy was not acceptable in many cases, $\{50 < N < 223\}$, it continues without being after the refinement. On the other hand, in Fig. 5 we illustrate the estimation of the number of contending UEs by (7) from $P_n(s, c)$; we observe that the initial results are very accurate, an
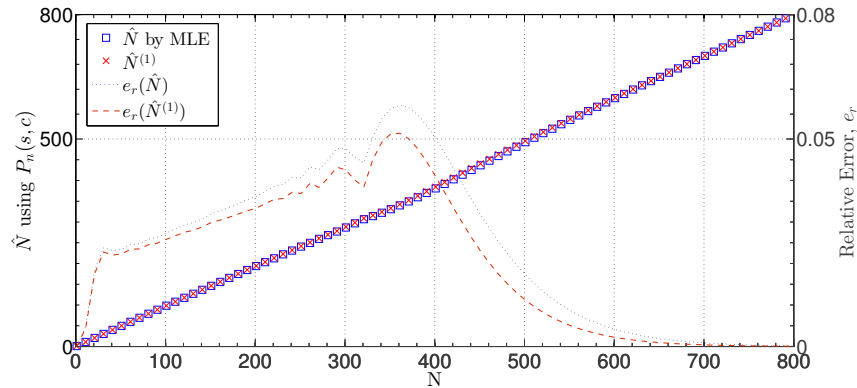
Fig. 5. MLE of contending UEs at eNB using $P_n(s,c)$, $r = 54$.

absolute error of 6% is obtained in the worst case. After refining these results by (8), we obtain an $e_r \leq 5\%$ for all the values of N tested.

5 Conclusions and Future Work

We presented an analytical approach to estimate the number of contending users in one random access slot for an LTE-A network. Based on the preamble information available at the evolved Node B, we find the conditional joint probability distribution function (PDF) for the number of successful and collided preambles within a random access slot. Then, we design a maximum likelihood estimator using this PDF. Numerical results showed that the proposed method can accurately estimate the number of contending MTC devices even for large network loads. This approach can further be extended to design static or dynamic congestion control schemes for alleviating the radio access network overload and optimize the parameter setting of these overload control schemes.

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