

Ellipses and ellipsoids: Matlab examples

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Code executed in Matlab R2023a

Presentations in video:

<http://personales.upv.es/asala/YT/V/ellip1EN.html> , <http://personales.upv.es/asala/YT/V/ellip2EN.html> , <http://personales.upv.es/asala/YT/V/ellip3EN.html> .

Objectives: support with Matlab code the understanding of the basic ideas about ellipses and ellipsoids.

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```
syms x [2 1] real
x
```

x =

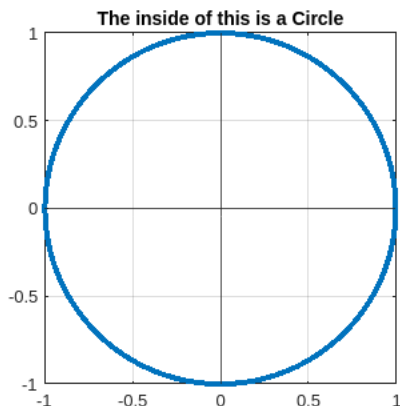
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Circumference

```
M=eye(2);
x'*M*x
```

ans = $x_1^2 + x_2^2$

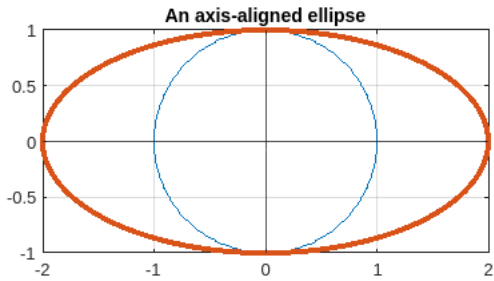
```
fimplicit(x'*M*x-1,LineWidth=3), grid on, axis equal
xline(0), yline(0), title("The inside of this is a Circle")
```



```
fimplicit (x'*M*x-1), grid on, axis equal %repeat unit circle, to compare
```

Axis-aligned ellipse

```
M=[1/2^2 0;0 1];
hold on
fimplicit(x'*M*x-1,LineWidth=3)
xline(0), yline(0)
hold off, title("An axis-aligned ellipse")
```



Generic ellipse (rotation matrix)

```
fimplicit(x'*eye(2)*x-1,'-.'), grid on, axis equal %repeat unit sphere
M=[1/2^2 0;0 1];
hold on
fimplicit(x'*M*x-1,'-.') %aligned with axis
alpha=30*pi/180;
R=[cos(alpha) -sin(alpha); sin(alpha) cos(alpha)] %Rotate alpha radians
```

```
R = 2x2
    0.8660    -0.5000
    0.5000     0.8660
```

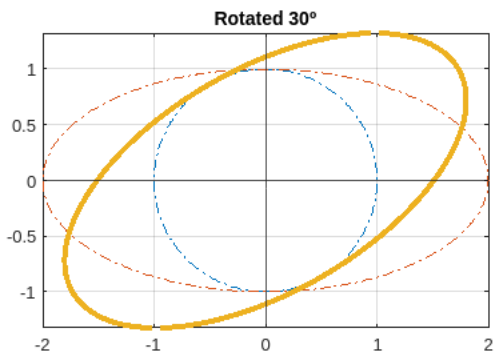
```
R'*R
```

```
ans = 2x2
     1     0
     0     1
```

```
P=R*M*R' %arbitrary positive-definite matrix
```

```
P = 2x2
    0.4375    -0.3248
   -0.3248     0.8125
```

```
fimplicit(x'*P*x-1,LineWidth=3)
hold off, title("Rotated 30°")
xline(0), yline(0)
```



Diagonalization

```
[V,D]=eig(P) %we recover "rotation" and "shape" matrices
```

```
V = 2x2
    -0.8660    -0.5000
    -0.5000     0.8660
D = 2x2
    0.2500     0
     0     1.0000
```

```
P-V*D*V'
```

```
ans = 2x2
10-15 ×
     0    -0.1110
    -0.1110     0
```

```
M
```

```
M = 2x2
    0.2500     0
     0     1.0000
```

```
R
```

```
R = 2x2
    0.8660    -0.5000
    0.5000     0.8660
```

```
atan(V(2,1)/V(1,1))*180/pi %rotation angle recovered
```

```
ans = 30.0000
```

3D ellipsoid visualization

```
%P=[3 2 1;2 0.5 2;1 2 5] %hyperboloid
%P=[3 2 1;2 1.714287 2;1 2 5] %cylinder (degenerate ellipsoid)
P=[3 2 1;2 1.9 2;1 2 5] %ellipsoid
```

```
P = 3x3
    3.0000    2.0000    1.0000
```

```

2.0000    1.9000    2.0000
1.0000    2.0000    5.0000

```

```
pei=eig(P)'
```

```

pei = 1x3
    0.1305    2.9009    6.8686

```

```
semiax=1./sqrt(pei)
```

```

semiax = 1x3
    2.7683    0.5871    0.3816

```

```

syms z [3 1] real
fimplicit3(z'*P*z-1,'EdgeColor','none','FaceAlpha',.8), axis equal
T=[eye(2) zeros(2,1)]

```

```

T = 2x3
    1     0     0
    0     1     0

```

```
Mproj12=inv(T*inv(P)*T')
```

```

Mproj12 = 2x2
    2.8000    1.6000
    1.6000    1.1000

```

```

hold on
fimplicit(x'*P(1:2,1:2)*x-1,'r',LineWidth=3)
%fimplicit(x'*Mproj12*x-1,'b',LineWidth=2.5)
hold off

```

