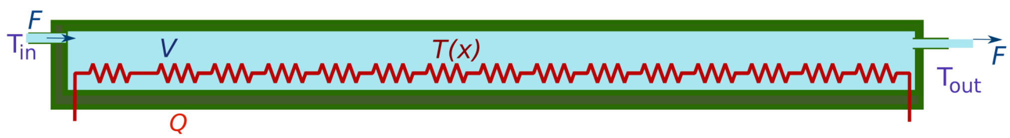
Tubular heater: simulation of PDE solution

Video-presentation: http://personales.upv.es/asala/YT/V/termedpstepEN.html

Objectives: Understand the PDE solution (step response).



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Modelling

Partial differential equation

This is the EDP, discussed in other materials:

$$\frac{\partial T}{\partial t} = -\frac{1}{S}F\frac{\partial T}{\partial x} - \frac{\bar{\kappa}}{S\rho C_e}T + \frac{\bar{Q}}{S\rho C_e}$$

Particular c ases of interest are:

- With $\bar{Q} = 0$ and $\bar{\kappa} = 0$ we get the 1D transport delay equation $\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial x}$, being $v = \frac{F}{S}$ the linear

transport speed.

- Steady state, $\frac{\partial T}{\partial t} = 0$; denoting the solution as $T_{eq}(x)$, we have $\frac{\partial T_{eq}}{\partial x} = -\frac{\bar{\kappa}}{F\rho C_e}T_{eq} + \frac{1}{F\rho C_e}\bar{Q}_{eq}(x)$.

WIth constant heating power along all the pipe length, defining $\lambda = \frac{\bar{\kappa}}{F\rho C_e}$, we get:

$$T_{eq}(x) = T_{in}\cdot e^{-\lambda\cdot x} + (1 - e^{-\lambda\cdot x})\bar{\kappa}^{-1}\cdot\bar{Q}_{eq}$$

Transfer function representation for constant flow (zero initial conditions)

We will work with the EDP

$$\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial x} - aT + b\bar{Q}$$

where $a = \dfrac{\bar{\kappa}}{S\rho C_e}$, $b = \dfrac{1}{S\rho C_e}$, $v = \dfrac{F}{S}$, to streamline notation. For constant flow, uniform heating power, solving the EDP via Laplace method, we get the transfer function (for each x):

$$\mathbb{T}(x,s) = (1 - e^{-\frac{s+a}{v}\cdot x})\frac{b}{(s+a)}\cdot\bar{\mathbb{Q}}(s) + \mathbb{T}_{in}(s)e^{-\frac{s+a}{v}\cdot x}$$

Steady state: if $\bar{Q}$ and T_{in} were constant, we would obtain steady-state equations replacing s by zero:

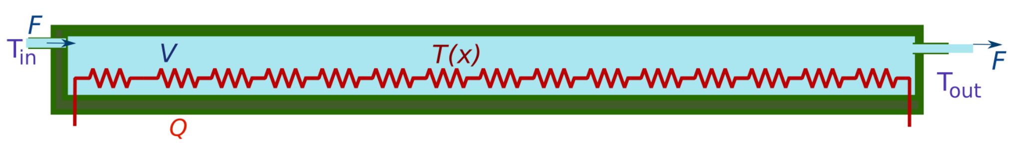
$T_{eq}(x) = (1 - e^{-\frac{a}{v}\cdot x})\dfrac{b}{a}\cdot\bar{Q}_{eq} + T_{in,eq}e^{-\frac{a}{v}\cdot x}$, obviously coincident with the formula above for this case.

Outlet temperature transient (Laplace domain):

Denoting $\phi = L/v$, we have:

$$\mathbb{T}_{out}(s) = \begin{pmatrix}(1 - e^{-\phi\cdot s}e^{-a\phi})\dfrac{b}{s+a} & e^{-\phi\cdot s}e^{-a\phi}\end{pmatrix}\cdot\begin{pmatrix}\bar{\mathbb{Q}}(s)\\ \mathbb{T}_{in}(s)\end{pmatrix}$$

where the coefficients as a function of physical parameters are $a = \dfrac{\bar{\kappa}}{S\rho C_e}$, $b = \dfrac{1}{S\rho C_e}$, $v = \dfrac{F}{S}$, $\phi = \dfrac{L}{v} = \dfrac{V}{F}$, and, hence, $a\phi = \dfrac{\bar{\kappa}L}{F\rho C_e} = \dfrac{\bar{\kappa}L}{vS\rho C_e}$



Exact simulation of the EDP solution (only outlet temp.)

Simulación exacta (control systems toolbox)

We'll set some values for physical parameters

```
S=0.0008;rho=1000;Ce=4180;barkappa=1.5e-1;
Ltot=2; %Total length (later on, we'll divide in several "finite elements" of 1st orde
F=0.00015; %cubic meters per second
```

So the coefficients of the transfer functions are:

```
b=1/S/rho/Ce*1e3 %we assume input in Kw per meter
```

```
b = 0.2990
```

```
a=b*barkappa
```

```
a = 0.0449
```

```
L=Ltot; %a single element with all of the heater's length
V=S*L %cubic meters
```

```
V = 0.0016
```

```
v=F/S %linear speed
```

```
v = 0.1875
```

```
phi=L/v%flushing time.
```

```
phi = 10.6667
```

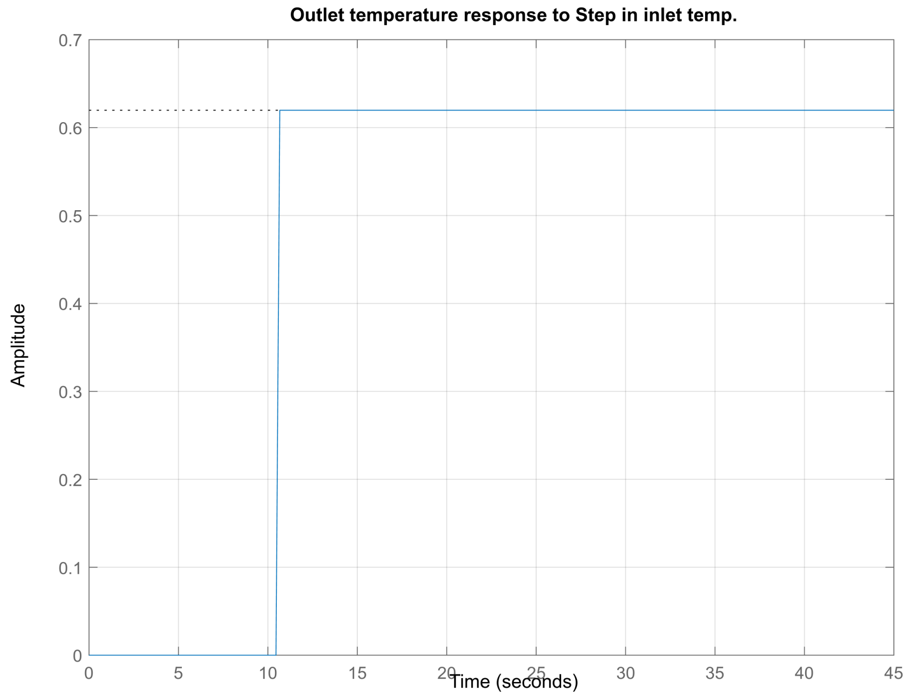
```
expphia=exp(-phi*a) %longitudinal temperature decay from PDE solution
```

```
expphia = 0.6197
```

Inlet Temperature step

It is just a static gain plus delay:

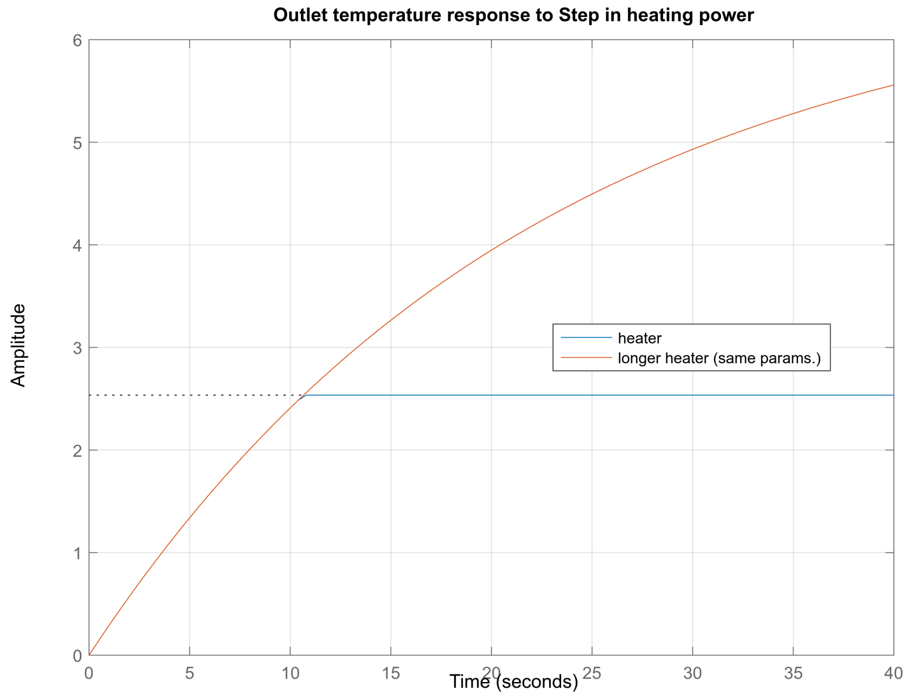
```
s=tf('s');
sysTin=exp(-a*phi)*exp(-phi*s);
sysQ=(1-sysTin)*b/(s+a);
sysPDE=[sysQ sysTin];
step(sysTin), grid on, title("Outlet temperature response to Step in inlet temp.")
```



Heating power step

It's a usual 1st-order step response but "truncated" when pipe ends and the fluid stops heating:

```
step(sysQ,b/(s+a),40), grid on, title("Outlet temperature response to Step in heating p
legend("heater", "longer heater (same params.)",Location="best")
```



We can simulate two particilar cases:

- lossless ($a = 0$): the outlet temperature is exactly T_{in} delayed by flushing time; a step in Q produces the step response of $\frac{1 - e^{-\phi s}}{s} b$ which is a ramp ascending during the flushing time with slope b and later on keeping the constant value ϕb.
- With losses ($a > 0$), what we presented above: delay plus gain in response to inle temperature and truncated 1st order response to step in Q, the response of $\frac{b}{s + a}$, truncated at $t = \phi$ constant from that instant at the value $\frac{b}{a}(1 - e^{-a\phi})$.