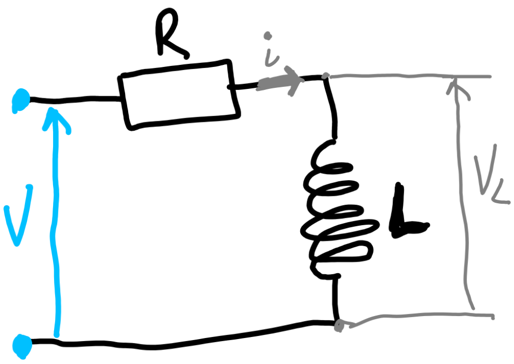
Applications of ODE: Inrush overcurrent in a linear inductor (I)



Presentation in video: https://personales.upv.es/asala/YT/V/LinrushlinEN.html

*Python code there as well.

See also https://youtu.be/x24545H5HdU?si=8KGq9tnQAULMkciZ from other channel.

```
L=1; R=1.5;
```

Differential equation:

$$\frac{di}{dt} = \frac{1}{L} \cdot (V - R \cdot i)$$

Normalized form $\dot{x} = Ax + Bu$:

$$\frac{di}{dt} = \frac{-R}{L} \cdot i + \frac{1}{L} \cdot V$$

```
t_final=2.2;
syms t real
syms i(t)
omega=50*2*sym(pi); %50 Hz AC
ph0=0*pi/2; %initial phase (we'll play with it later on)
%ph0=-phaseI_ss;
V_peak=120;
V(t)=V_peak*sin(omega*t+ph0) %input voltage
```

V(t) = $120 \sin(100\,\pi\, t)$

```
vpa(V(t),4)
```

ans = $120.0 \sin(314.2\, t)$

Let us solve $\frac{di}{dt} = \frac{1}{L} \cdot (V - R \cdot i)$:

```
sol=dsolve( diff(i)==(V(t)-R*i(t))/L , i(0)==0) %ODE symbolic solution
```

sol =

$$\frac{120\left(\frac{3\sin(100\,\pi\,t)}{2}-100\,\pi\cos(100\,\pi\,t)\right)}{10000\,\pi^2+\frac{9}{4}}+\frac{48000\,\pi\,\mathrm{e}^{-\frac{3\,t}{2}}}{40000\,\pi^2+9}$$

```
vpa(sol,4)
```

ans = $0.001824\sin(314.2\,t)-0.382\cos(314.2\,t)+0.382\,\mathrm{e}^{-1.5\,t}$

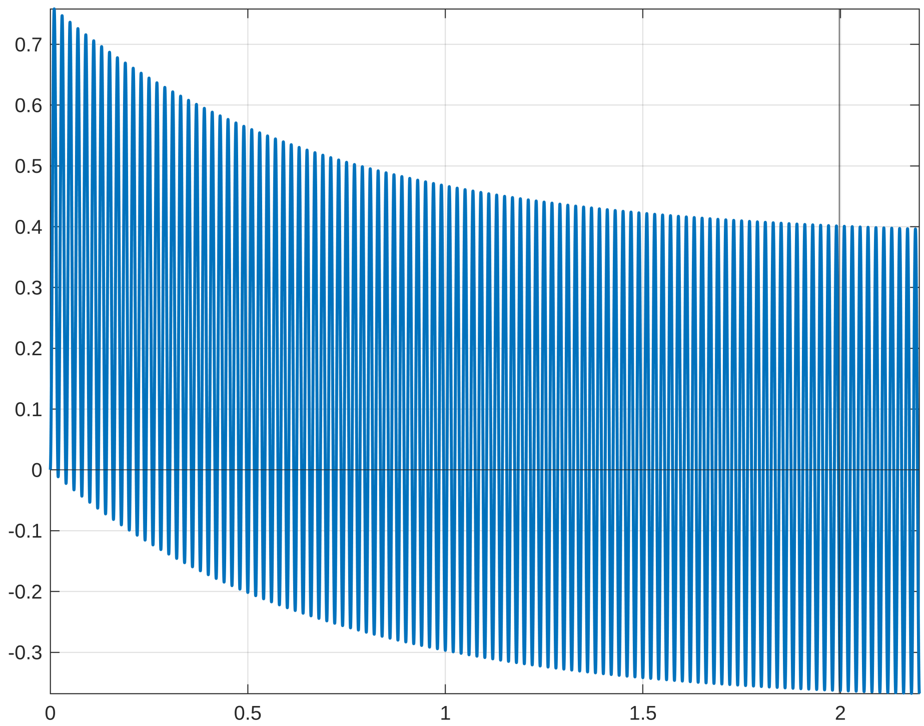
```
circuit_pole=-R/L %coefficient of the exponential
```

```
circuit_pole =
-1.5000
```

```
t_settling=fzero(@(t) exp(circuit_pole*t)-0.05,0) % 5 percent settling time
```

```
t_settling =
1.9972
```

```
fplot(sol,[0 t_final],LineWIdth=1.5), grid on, yline(0), xline(t_settling)
```



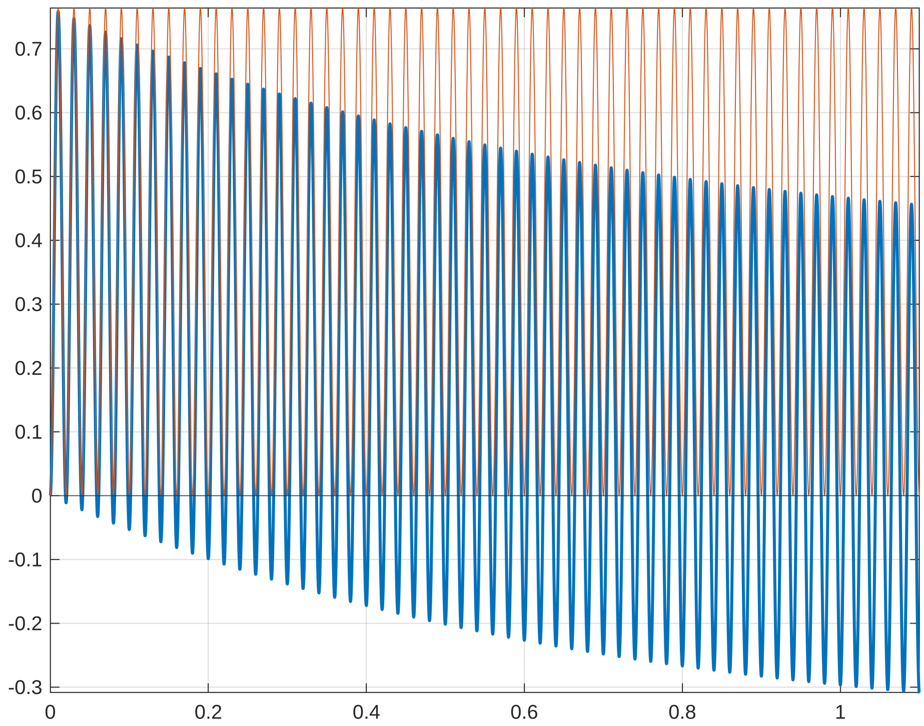
No resistor particular case

In this case $\frac{di}{dt}=\frac{1}{L}V$; hence $i(t)=i(0)+\frac{1}{L}\cdot\int_0^t V(\tau)\,d\tau$

```
syms tau real
i_noR=int(V(tau)/L,0,t)+0;
vpa(i_noR,4)
```

ans = $0.7639 \sin(157.1\, t)^2$

```
figure()
fplot(sol,[0 t_final/2], LineWidth=1.5), hold on
fplot(i_noR,[0 t_final/2]), hold off
grid on, yline(0)
```



Particular case 2: steady state sinusoidal regime

```
Z=R+L*omega*1j %impedance
```

Z =

$$\frac{3}{2} + 100\,\pi\,\mathrm{i}$$

```
amplI_ss=abs(1/Z)*V_peak; %Current amplitude
eval(amplI_ss)
```

```
ans =
0.3820
```

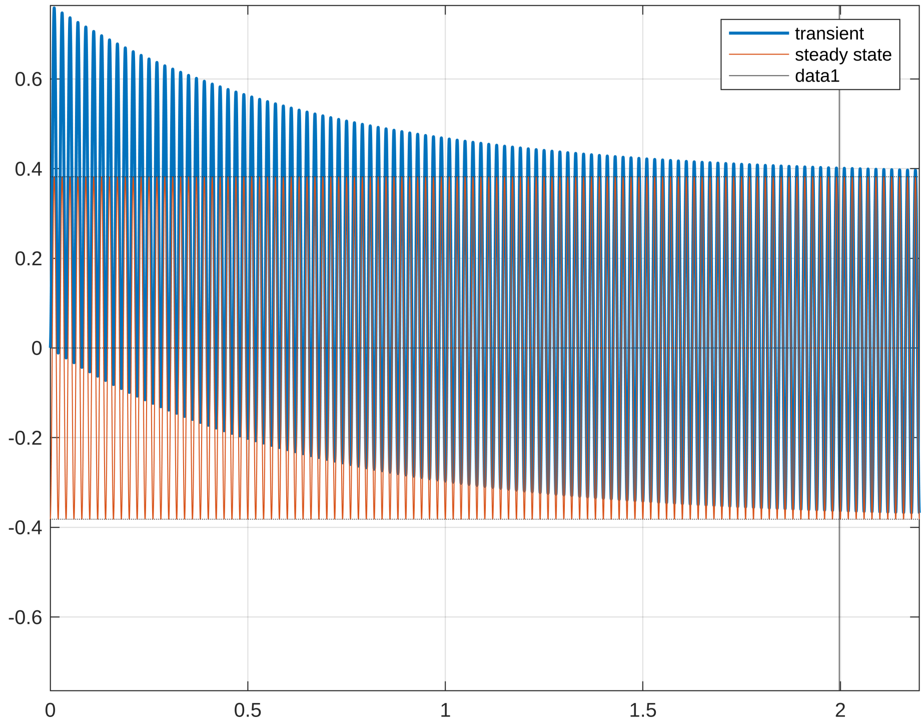
```
phaseI_ss=angle(1/Z);
eval(phaseI_ss)*180/pi
```

```
ans =
-89.7264
```

```
I_ss=simplify(amplI_ss*sin(omega*t+phaseI_ss+ph0),120);
vpa(I_ss,4)
```

ans = $-0.382\sin(1.566 - 314.2\,t)$

```
figure()
fplot(sol,[0 t_final],LineWidth=1.5), hold on
fplot(I_ss,[0 t_final]), hold off
yline([-2 -1 0 1 2]'*eval(amplI_ss),':'), grid on
legend("transient", "steady state",
Location="best"),xline(t_settling)
```



Conclusions

Initial response close to "no resistor" case, which has a non-vanishing DC component.

- If $V(0) = 0$, we have twice initial current to that of steady state from impedance computations (2x inrush current); it takes $\approx 3\frac{L}{R}$ seconds to vanish.

- If phase of voltage equal to that of conductance (inverse impedance) then we start "spot on" the steady-state sinusoid, inrush current avoided. It is close to 90^o for a heavily inductive load.

- ***Inrush current*** in real transformers may be much higher (6x or more), due to magnetic saturation (in another video). Harmonics (non-sinusoidal waveforms) also appear in such a case. Fuse ratings, energization timing, serial resistance, etc. must be considered with care in, say, high-power transformer startup.