Stability margins and worst-case gains (2nd order open-loop example): usage of robust control toolbox commands

This code executed without errors in Matlab **R2022b**

Video-presentation at: http://personales.upv.es/asala/YT/V/stabmrgEN.html

Whole collection at: http://personales.upv.es/asala/YT/

Objectives: understand the usage of **robstab**, **wcgain** and **robgain** in a simple uncertain 2nd order system (may be understood as a mass-spring-damper with uncertain damping).

Model

System model will be $y(s) = \frac{5}{s^2 + (1 + 0.5\delta) \cdot s + 5} \cdot u(s)$, with $\delta \in \mathbb{R}$, actually with $-1 < \delta < 1$, so we already normalised the damping uncertainty. Parametric uncertainty is coded as:

```
delt=ureal("Delta",0);
```

Given that the transfer function $\frac{a}{s^2 + bs + c}$ can be modelled in state-space as

A=[0 1;-c -b],   B=[0;a],   C=[1 0],   D=0

we may understand the model as the uncertain state-space $\dot{x} = Ax + Bu$, $y = Cx + Du$ with A, B, C, D given as:

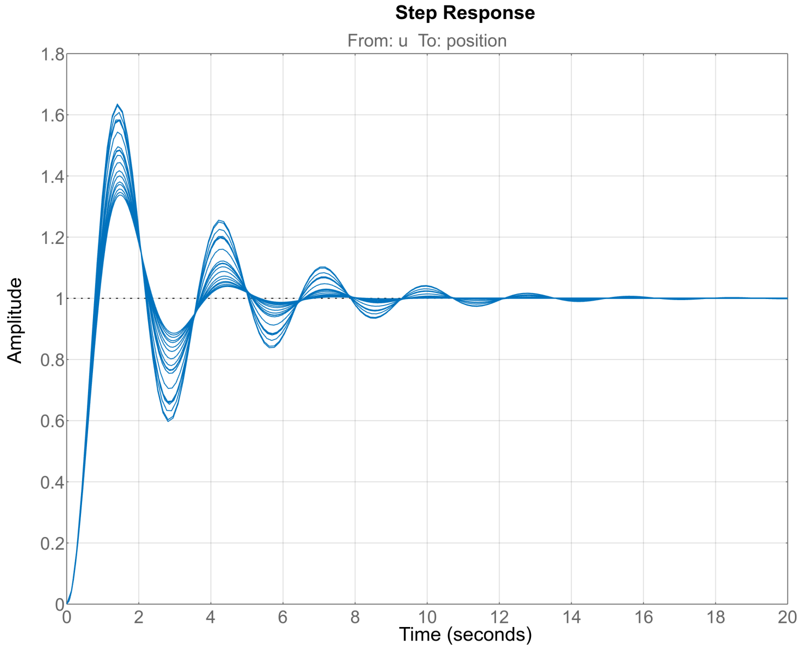
```
A=[0 1;-5 -(1+0.5*delt)]; %matrix with uncertain elements
%This uncertain matrix syntax would not work if "delt" were ultidyn
%modelling, for instance, an uncertain damper with dynamics inside.
% In such a case, we would need some system interconnection commands
% such as LFT, or a block-diagram representation with connect, etc.
% This is discussed in other modelling examples of my teaching materials.
GsUnc=ss(A,[0;5],[1 0],0);
GsUnc.InputName={'u'};
GsUnc.OutputName={'position'}
```

```
GsUnc =

  Uncertain continuous-time state-space model with 1 outputs, 1 inputs, 2 states.
  The model uncertainty consists of the following blocks:
    Delta: Uncertain real, nominal = 0, variability = [-1,1], 1 occurrences

Type "GsUnc.NominalValue" to see the nominal value, "get(GsUnc)" to see all properties, and "GsUnc.Uncerta
```

```
step(GsUnc), grid on
```



Damping uncertainty translates to settling time and overshoot uncertainty, for instance.

Nominal model versus worst-case models

```
G_nominal=GsUnc.NominalValue;
zpk(G_nominal) %this is not "uncertain"
```

```
ans =

  From input "u" to output "position":
        5
  -------------
  (s^2 + s + 5)

Continuous-time zero/pole/gain model.
```

Nominal resonance peak will be the "infinity norm":

```
[nominal_peakgain,freq]=norm(G_nominal,inf)
```

```
nominal_peakgain = 2.2942
freq = 2.1225
```

Robustness analysis (uncertain model)

```
[rbm,wcus]=robstab(GsUnc)
```

```
rbm = struct with fields:
           LowerBound: 2.0000
           UpperBound: 2.0000
    CriticalFrequency: 2.2361
wcus = struct with fields:
```

```
    Delta: -2.0000
```

```
G_unstable=usubs(GsUnc,wcus);tf(G_unstable)
```

```
ans =
 
  From input "u" to output "position":
           5
  --------------------
  s^2 + 2.22e-16 s + 5
 
Continuous-time transfer function.
```

```
[wcg,wcug]=wcgain(GsUnc)
```

```
wcg = struct with fields:
           LowerBound: 4.5004
           UpperBound: 4.5080
    CriticalFrequency: 2.2079
wcug = struct with fields:
    Delta: -1
```

```
G_worstcaseperf=usubs(GsUnc,wcug); tf(G_worstcaseperf)
```

```
ans =
 
  From input "u" to output "position":
         5
  ---------------
  s^2 + 0.5 s + 5
 
Continuous-time transfer function.
```

```
limgain=2.8;
[rbg,wcr]=robgain(GsUnc,limgain) %maximum "delta" to avoid hitting the peak
gain limit
```

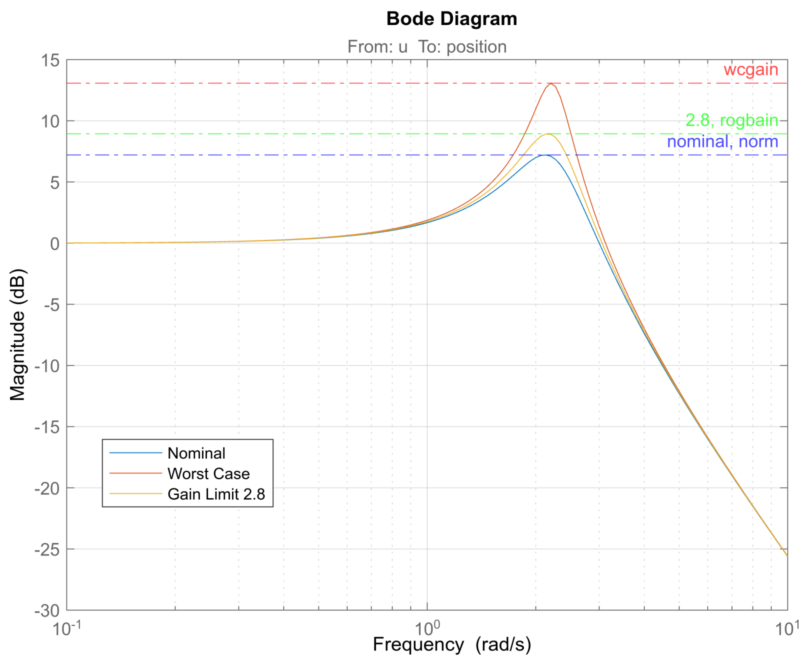
```
rbg = struct with fields:
           LowerBound: 0.3750
           UpperBound: 0.3758
    CriticalFrequency: 2.1613
wcr = struct with fields:
    Delta: -0.3758
```

```
G_limgain=usubs(GsUnc,wcr); tf(G_limgain)
```

```
ans =
 
  From input "u" to output "position":
          5
  ------------------
  s^2 + 0.8121 s + 5
 
Continuous-time transfer function.
```

Bode magnitude plots to check results

```
bodemag(G_nominal,G_worstcaseperf,G_limgain,logspace(-1,1,150)), grid on
yline(20*log10(wcg.UpperBound),'-.r',Label="wcgain")
yline(20*log10(nominal_peakgain),'-.b',Label="nominal, norm")
yline(20*log10(limgain),'-.g',Label="2.8, rogbain")
legend("Nominal","Worst Case","Gain Limit 2.8", Location="best")
```




```
%hold on
%bodemag(GsUnc)
%hold off
```