Aircraft Longitudinal Dynamics: “Phugoid” mode (simplified equations)

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Link to materials, comments, etc. in description and personal website.
Video presentation at: http://personales.upv.es/asala/YT/V/fugoid1EN.html

Outline

Motivation:

Gliders "glide" around a certain angle. Fixed-wing drones too. An airplane model is "complicated" (6 DoF, aerodynamics, aeroelasticity, control surfaces)... Sometimes, a "simple" model that "comprehensibly" explains something, even if it's crude approximation, can serve to learn/draw conclusions.

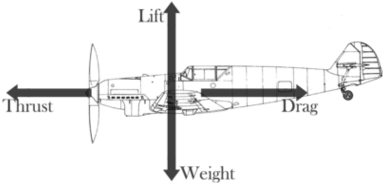
Objectives:

Model, in a "very simplified" way, the glide of an aircraft ("phugoid" mode in the literature).

Contents:

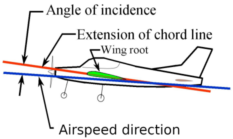
Approximate assumptions. Intrinsic coordinates. Balance of forces. State equations. Conclusions.

"Longitudinal" forces in an aircraft

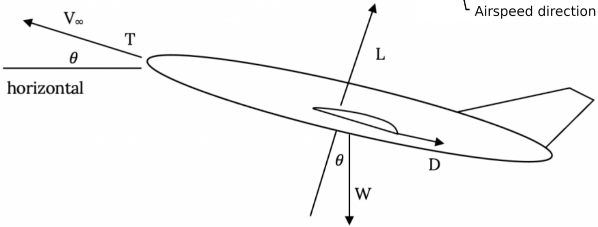


[level flight, $\theta = 0$]

*We do not consider aerodynamic torque or offset forces with respect to CoG.



[climb, $\theta > 0$]



Why intrinsic coordinates?

- Lift (L) and drag (D) are by definition "normal" and "tangential" respectively. Propulsion is roughly tangential.

- The pilot "feels" the plane's forward speed, pitch, and changes in the plane's orientation, $\frac{d\theta}{dt}$ (the angular velocity that a gyroscope would measure).

- It therefore seems recommendable to work in "tangent/normal" so that only the weight has trigonometric formulas, and try to make the angle θ itself a state variable.

Problem 1: as the frame of reference is not "inertial", one must be careful.

Problem 2: "body" frame $\neq$ "path" frame; we'll assume they are the same, smooth maneuver, constant angle of incidence.

Longitudinal phugoid mode assumptions

- Planar (x, y) movement, longitudinal displacement vs. elevation.
- Negligible effect of Moment of Inertia. The aerodynamics aligns it "infinitely fast" at a fixed angle α to the airspeed (the plane behaves like a weathervane).
- Negligible aircraft length with respect to trajectory curvature radius.

In other words, a 2GL dynamic is assumed, where the angle depends on the trajectory (x, y), order 4 in positions, order 2 in speeds.

***Formulae from Zhukovski (1891)-Lanchester (1908).**
https://en.wikipedia.org/wiki/Nikolay_Zhukovsky_(scientist)

The angle of incidence α is assumed to be constant (it would depend on control surfaces, thrust... but they will NOT do so in the simplified eqs.). The plane is instantly aligned with the "airspeed" vector; it behaves like a "weathervane", at the angle that makes the "resultant torque" zero.

Kinematics in intrinsic coordinates (tangent/normal)

Tangent Vector:

$$\vec{v}(t) := \nu(t) \cdot \vec{T}(t),$$
$$\vec{T}(t) = (\cos\theta(t), \sin\theta(t)),$$

- we define $\vec{T}$ as the direction of speed vector ($\vec{T}$ is a unit vector).
- Linear velocity ν will be the "airspeed" of the plane's center of gravity.

Left normal vector: (counterclocwise rotation from direction of movement)

$$\frac{d\vec{T}}{dt} = (-\sin\theta, \cos\theta) \cdot \frac{d\theta}{dt} := \frac{d\theta}{dt}\vec{N}_L, \text{ perp. to } \vec{T}.$$

Dynamics in tangential/normal coordinates

Acceleration in intrinsic coordinates:

$\frac{d\vec{v}}{dt} = \frac{d\nu}{dt}\vec{T} + \nu\frac{d\vec{T}}{dt}$, Hence, $\frac{d\vec{v}}{dt} = \frac{d\nu}{dt}\vec{T} + \nu\frac{d\theta}{dt}\vec{N}_L$

We define normal and tangential acceleration:

$$a_T = \frac{d\nu}{dt}, \qquad a_{N_L} = \nu\frac{d\theta}{dt}$$

Incorporating forces (constant mass assumed):

$$m \cdot \frac{d\nu}{dt} = F_T, \qquad m \cdot \nu \cdot \frac{d\theta}{dt} = F_{N_L}$$

*from now on, we forget about $\vec{v}$, and we'll use v instead of ν for the scalar "airspeed".

Phugoid mode equations of an aircraft

Tangential dynamics:

$$\frac{dv}{dt} = \frac{1}{m}F_{res,T} = \frac{1}{m}(-mg\sin\theta - Drag + Thrust)$$

Normal dynamics:

$$v\frac{d\theta}{dt} = \frac{1}{m}F_{res,T} = \frac{1}{m}(-mg\cos\theta + Lift)$$

*If thrust did not have an exactly tangential resultant (by construction or because guidance is done with *thrust vectoring*, changing γ below), it would be:

$$\frac{dv}{dt} = \frac{1}{m}(-mg\sin\theta - Drag + Thrust\cdot\cos\gamma)$$
$$v\frac{d\theta}{dt} = \frac{1}{m}(-mg\cos\theta + Lift + Thrust\cdot\sin\gamma)$$

Phugoid mode equations (normalized internal repr.)

Replacing $Lift = l \cdot v^2$ and $Drag = d \cdot v^2$ with expressions that, for simplicity, we only make dependent on v^2 (actually, also dependent on the orientation of the aircraft with respect to airspeed velocity vector, ...), we have:

$$\frac{dv}{dt} = -g \sin\theta - \frac{d}{m} \cdot v^2 + \frac{1}{m} u$$

$$\frac{d\theta}{dt} = -g \frac{\cos\theta}{v} + \frac{l}{m} \cdot v$$

$$\frac{dx}{dt} = v \cdot \cos\theta$$

$$\frac{dy}{dt} = v \cdot \sin\theta$$

With this state equation, we can now carry out simulations, e.g. with Matlab `ode45`.

*If v approaches 0, we would have a singularity.

Conclusions

- Longitudinal flight "if moment of inertia is small" or/and "if the corrective torque against angular deviation is large" can approximate to 2DoF, order 4 model or order 2 model in speeds. The plane as a "point mass".
- In intrinsic coordinates (tangential and normal), we have obtained a model with linear (tangential) and angular velocities as state variables (apart from positions, of course).
- Angular velocity is that of the aircraft's CoG path, under certain assumptions... incorporating "body frame" requires moment of inertia, resultant torque, and the variation of thrust/drag with angle of incidence ("body" orientation- "COD trajectory" orientation). Not discussed here for brevity.

