Theory of Distributions. Applications to Physics and Engineering

2009-2010

Number of credits: 3 (30 hours).
Department: Applied Mathematics.
Instructor: Vicente Montesinos Santalucía

1 Introduction

This is an intermediate course in distributions with applications to Physics and Engineering. It is addressed to students and teachers wishing to have a deeper insight in the theory of heat and waves. The background required is a course in mathematical analysis and one in elementary topology. All the course will be taught in English.

The Dirac's δ associates to a continuous periodic function its value at zero. In a framework more general than the usual differential calculus, the Dirac's δ can be differentiated, even it is infinitely differentiable. A continuous function can not, in general, be differentiated. However, in this context, it has also derivatives of any order. The advantages of this operational calculus in solving ordinary and partial differential equations is plain: it is possible to solve the classical problems (heat, wave, Laplace equations and the Dirichlet problem) in more general situations, with a conceptually more simple approach.

2 Grading

50% of the grades: solving the assignments proposed by the instructor. 50%: a final written or oral exam, or by a written report on the subject.

3 Background needed

A second course in Mathematical Analysis (like the one corresponding to the third semester in Telecommunication Engineering) or equivalent.

4 Program

1. Periodic functions and periodic distributions.
2. Fourier series of smooth periodic functions and periodic distributions.
3. The heat equation: distribution and classical solutions.
4. The wave equation.
5. Laplace's equation and the Dirichlet problem.
6. The Laplace transform. The spaces $\mathcal{L}^2$ and $\mathcal{L}^1$. Characterizations of distributions of type $\mathcal{L}^1$. Laplace transform of functions and distributions.
7. Applications to differential equations.

5 Bibliography

1. R. Beals. *Advanced Mathematical Analysis*. GTM 12. Springer-Verlag, 1973.
2. W. Rudin. *Functional Analysis*. McGraw Hill. 1973.
3. P. R. Wallace. *Mathematical Analysis of Physical Problems*. Dover, 1984.