ABOUT USING THE MINIMUM ENERGY DISSIPATION TO FIND THE STEADY-STATE FLOW DISTRIBUTION IN NETWORKS

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Introduction

Our main interest: HVAC air duct return and supply networks.

General goal (ongoing research)

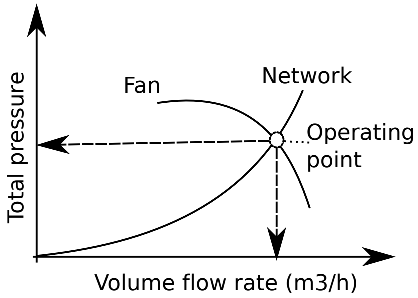
To devise *new method*, based on the **Principle of Minimum Energy Dissipation**, to analyse the steady-state flow distribution within a network, *which deals "naturally" with branched junctions.* (Incompressible and practically isothermal flows)

Particular goal (HEFAT2019)

To extend the previous results for tree-shaped networks (see [1]), to general ones, **but here the branching effects are neglected**.

Why a new method?...

System resistance - fan curve:



Assume: pressure drop at j-section (or at a single element):

- volume flow rate $\dot{V}$
- loss coefficient $\widehat{K}_j$

$$\Delta p_j = \hat{K}_j \cdot v^2 = \hat{K}_j \cdot \frac{\dot{V}_j^2}{A_j^2} \qquad (1)$$

System resistance by electrical analogy:

Equivalent loss coefficient $\hat{K}_{eq}$

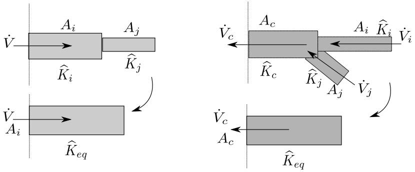
$$\widehat{K}_{eq} = \widehat{K}_i + \widehat{K}_j \cdot \left(\frac{A_i}{A_j}\right)^2 \quad (2)$$

$$\widehat{K}_{eq} = \widehat{K}_c + H_{i,j}^2 \cdot \widehat{K}_j \cdot \left(\frac{A_c}{A_j}\right)^2 \quad (3)$$

where:

$$H_{i,j} = \left[1 + \frac{A_i}{A_j} \cdot \sqrt{\left(\frac{\widehat{K}_j}{\widehat{K}_i}\right)}\right]^{-1} \quad (4)$$

Series Y

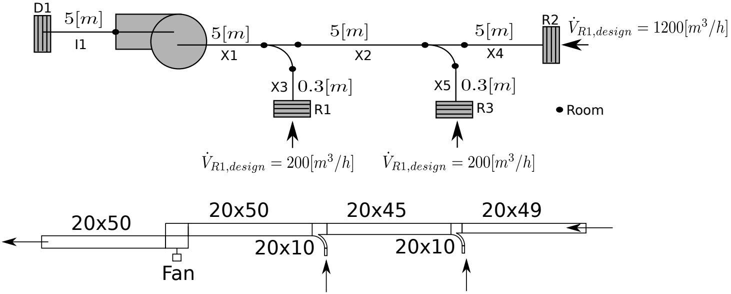


Remark:
$\hat{K}_j$ is a function of the flow rate or even of flow rate ratios (branched junctions)

Calculation of the operating point

1. START: Guess a flow distribution. Compute hydraulic resistances.
2. COMPRESSION: Compute the duct-network characteristic curve ($\hat{K}_{eq,net}$) for that **guessed flow distribution**
3. INTERSECTION: between system and fan curves.
4. DECOMPRESSION: Re-compute the **new flow distribution** (at sections and diffusers and/or return grilles).
5. RETURN to step 2 until: new $\approx$ guessed flow distribution.

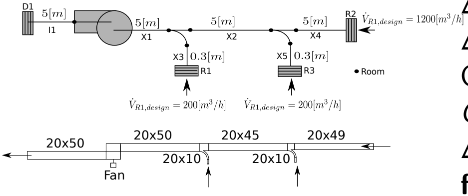
Example: sizing using equal friction method $\Delta p = 1[Pa/m]$



Balancing (design flows) :

- X1-X3 path: a loss of 21,15[*Pa*] is needed: cross sectional area reduction ratio of 0,56
- X1-X2-X5 path: a loss of 12[*Pa*] is needed: cross sectional area reduction ratio 0,61.

... design summary:



$\Delta p_{supply} = 36{,}72[Pa]$
$\Delta p_{return} = 23{,}74[Pa]$
Operating point:
$\dot{Q} = 1600[m^3/h]$
$\Delta p_{total} = 60{,}46[Pa]$
flow distribution = design.

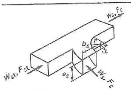
BUT, ...are the dampers really needed?

- What happens to the flow rates if the dampers are removed?

...to simplify, let's assume fan always delivers $1600[m^3/h]$.

Converging smooth wye ($r/b_0 = 1.0$) of the type $F_s + F_{st} \geqslant F_c$ of rectangular cross section; $\alpha = 90°$ [23, 27]

Diagram 7-11



Side branch

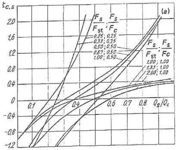
$$\zeta_{c.s} = \frac{\Delta p_s}{\rho w_c^2/2} = a_1 \left(\frac{Q_s}{Q_c}\right)^2 + b_1 \frac{Q_s}{Q_c} + c_1$$

see graph a; for a_1, b_1, and c_1, see the table

$$\zeta_s = \frac{\Delta p_s}{\rho w_s^2/2} = \frac{\zeta_{c.s}}{(Q_s F_c/Q_c F_s)^2}$$

Values of $\zeta_{c.s}$

$\frac{F_s}{F}$ $\left(\frac{F_s}{F_c}\right)$	Q_s/Q_c 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.25 (0.25)	−0.50	0	0.50	1.20	2.20	3.70	5.80	8.40	11.40
0.33 (0.25)	−1.20	−0.40	0.40	1.60	3.00	4.80	6.80	8.90	11.00
0.50 (0.50)	−0.50	−0.20	0	0.25	0.45	0.70	1.00	1.50	2.00
0.67 (0.50)	−1.00	−0.60	−0.20	0.10	0.30	0.60	1.00	1.45	2.00
1.00 (0.50)	−2.15	−1.45	−0.95	−0.50	0	0.40	0.80	1.30	1.90
1.00 (1.00)	−0.60	−0.30	−0.10	−0.04	0.13	0.21	0.29	0.36	0.42
1.33 (1.00)	−1.20	−0.80	−0.40	−0.20	0	0.16	0.24	0.32	0.38
2.00 (1.00)	−2.10	−1.40	−0.90	−0.50	−0.20	0	0.20	0.25	0.30



Straight passage

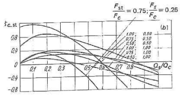
$$\zeta_{c.st} = \frac{\Delta p}{\rho w_c^2/2} = a_2 \left(\frac{Q_s}{Q_c}\right)^2 + b_2 \frac{Q_s}{Q_c} + c_2$$

See graph b; for a_2, b_2, and c_2, see the table

$$\zeta_{st} = \frac{\Delta p_{st}}{\rho w_{st}^2/2} = \frac{\zeta_{c.st}}{(1 - Q_s/Q_c)^2 (F_c/F_{st})^2}$$

Values of $\zeta_{c.st}$

$\frac{F_{st}}{F_c}$ $\left(\frac{F_s}{F_c}\right)$	Q_s/Q_c 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.75 (0.25)	0.30	0.30	0.20	−0.10	−0.45	−0.92	−1.45	−2.00	−2.60
1.00 (0.50)	0.17	0.16	0.10	0	−0.08	−0.18	−0.27	−0.37	−0.46
0.75 (0.50)	0.27	0.35	0.32	0.25	0.12	−0.03	−0.23	−0.42	−0.58
0.50 (0.50)	1.15	1.10	0.90	0.65	0.35	0	−0.40	−0.80	−1.30
1.00 (1.00)	0.18	0.24	0.27	0.26	0.23	0.18	0.10	0	−0.12
0.75 (1.00)	0.75	0.36	0.38	0.35	0.27	0.18	0.05	−0.08	−0.22
0.50 (1.00)	0.80	0.87	0.80	0.68	0.55	0.40	0.25	0.08	−0.10



F_s/F_{st}	F_{st}/F_c	F_s/F_c	a_1	b_1	c_1	a_2	b_2	c_2
0.25	1.00	0.25	19.82	−5.27	−0.03	−4.38	0.65	0.32
0.33	0.75	0.25	11.43	3.97	−1.76	−0.45	−0.39	0.22
0.50	1.00	0.50	1.96	1.04	−0.58	−1.70	0.57	0.25
0.57	0.75	0.50	1.96	1.64	−1.05	−2.68	0.42	1.23
1.00	0.50	0.50	−0.63	5.55	−2.65	−1.27	0.88	0.11
1.00	1.00	1.00	−1.55	2.88	−0.90	0.16	−1.30	0.84
1.33	0.75	1.00	−2.64	4.54	−1.60	−2.68	−0.42	1.23
2.00	0.50	1.00	−4.46	−7.41	−2.77			

Compression/Decompression + initial guess = design ...

it just crashes. **Why?**

A bad initial guess might produce a negative hydraulic resistance.

But a solution.., does exist!:

The algorithm must be modified but not in an obvious way.

Example:

design → actual solution:

- D1: 1600 → 1600 $[m^3/h]$
- R2: 1200 → 897 $[m^3/h]$
- R1: 200 → 414 $[m^3/h]$
- R3: 200 → 289 $[m^3/h]$

In short...

- some difficulties triggered this research.
- Additionally,... other design methods based on this compression/decompression (like T-method) show also difficulties.

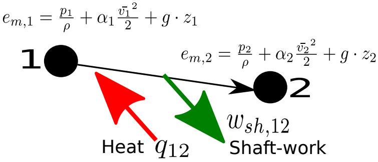
...thus, since the dissipated energy is always a positive quantity:

Could it alternatively be used to solve the problem?

Energy Dissipation

Energy Balance per unit mass (Extended Bernoulli equation):

$$(u_2 - u_1) + (\frac{p_2}{\rho} - \frac{p_1}{\rho}) + (\frac{\alpha_2 \bar{v}_2^2}{2} - \frac{\alpha_1 \bar{v}_1^2}{2}) - g(z_2 - z_1) = -w_{sh,12} + q_{12}$$



Specific dissipation
$\varphi[J/kg]$, $\widehat{\varphi}[J/m^3] \equiv [Pa]$:

$$e_{m,2} - e_{m,1} = -w_{sh,12} - \varphi_{12}$$
$$u_2 - u_1 = q_{12} + \varphi_{12}$$

Dissipation rate $\dot{\Phi}[W]$:
$\dot{\Phi} = \varphi \cdot \dot{m}$

Mechanical Energy Dissipation related to ...
Entropy generation:

$$\varphi = T \cdot \dot{S}_{gen} / \dot{m}$$

$$\dot{\Phi} = T \cdot \dot{S}_{gen} = \varphi \cdot \dot{m} = \varphi \cdot \rho \dot{V} = \widehat{\varphi} \cdot \dot{V}$$

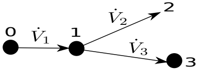
Darcy's friction factor f_D (Herwig et al.) for a straight conduit:

$$\widehat{\varphi} = f_D \cdot \frac{L_{12}}{D_h} \cdot \frac{\bar{v}^2}{2} \cdot \rho$$

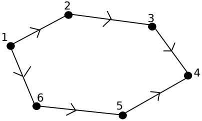
Pressure drop:

$$\Delta p_{12} = p_1 - p_2 = \widehat{\varphi}_{12}$$

Conventional methods:

- Mass conservation (at nodes): 

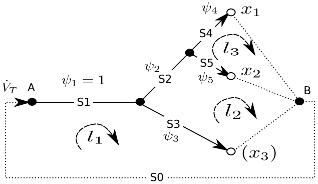
- **Energy conservation** (for each independent loop):



$$\widehat{\varphi}_{12} + \widehat{\varphi}_{23} + \widehat{\varphi}_{34} - (\widehat{\varphi}_{54} + \widehat{\varphi}_{65} + \widehat{\varphi}_{16}) = 0$$

Main question (see details in reference [1])

Can, latter equation, be obtained for each independent loop of a flow network, by using **the energy dissipation minimisation**?

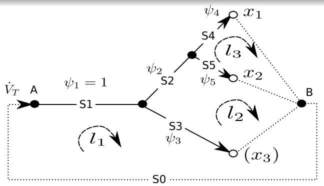


Network energy dissipation expression with flow constraint g:

$$\dot{\Phi} = \varphi_T \cdot \dot{m}_T = (\varphi_T \cdot \rho) \cdot \dot{V}_T = \widehat{\varphi}_T \cdot \dot{V}_T = \sum_{j=1}^{nsect} \widehat{\varphi}_j \cdot \dot{V}_j$$

$$\widehat{\varphi}_T = F(\psi_1, \ldots, \psi_{nsect}) = \sum_{j=1}^{nsect} \widehat{\varphi}_j \cdot \psi_j \quad \text{where; } g : \vec{x} \to \vec{\psi} \quad ; \psi_j = \frac{\dot{V}_j}{\dot{V}_T}$$

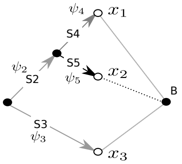
$$(F \circ g)(x_1, \ldots, x_{n-1}) = F(\psi_1(x_1, \ldots, x_{n-1}), \psi_2(x_1, ..), \ldots, \psi_{nsect}(x_1, ..))$$

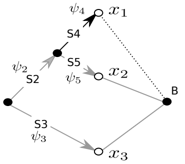


Network dissipation stationarity condition:

$$\mathcal{D}(F \circ g) = (\mathcal{D}F \circ g) \cdot \mathcal{D}g = \vec{0}$$

$$g: \quad \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right)$$

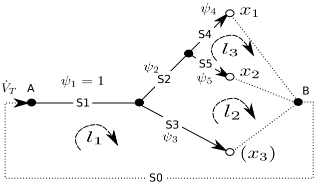




$$\mathcal{D}g = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathcal{D}(F \circ g) = (\mathcal{D}F \circ g) \cdot \mathcal{D}g = \vec{0} \tag{5}$$

..., what about the other term $(\mathcal{D}F \circ g)$?. Has any special form?



For the example of the Figure,... each component of eq. (5) must be equal to following energy balances:

$$\widehat{\varphi}_{S2} + \widehat{\varphi}_{S4} - \widehat{\varphi}_{S3} = 0$$
$$\widehat{\varphi}_{S2} + \widehat{\varphi}_{S5} - \widehat{\varphi}_{S3} = 0$$

However, in order to get this result, the **dissipation function** $\widehat{\varphi}$ **must have a definite form** which has to do with $(\mathcal{D}F \circ g)$

Conditions which ensure that the energy dissipation minimisation is equivalent to the energy balance of the independent loops. (see [1])

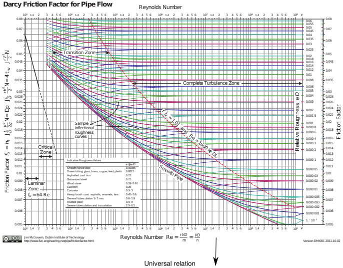
$\widehat{\varphi}_j = \widehat{K}_j \cdot \psi_j^m$
$\widehat{K}_j > 0$, and constant
$m \in \{\mathbb{R} - [-1, 0]\}$ and must be the same for all the network.

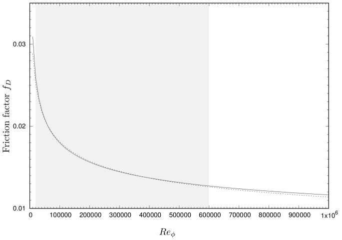
Note: in [1] we additionally showed, that the stationary point is a minimum.

In short the specific energy dissipation function (whose minimisation is equivalent to an energy balance) is:

$$F(\psi_1, \ldots, \psi_{nsect}) = (F \circ g)(x_1, \ldots, x_n) = \sum_{j=1}^{nsect} \widehat{K}_j \cdot \psi_j^{m+1} > 0$$

What is the physical meaning of $\widehat{K}_j$?, and the role of m? (Pipes)

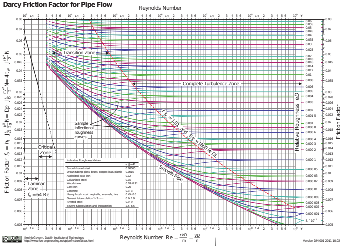


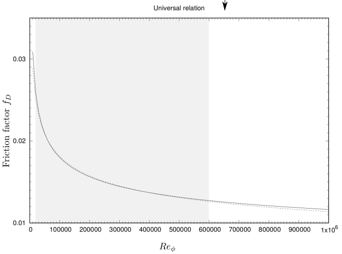


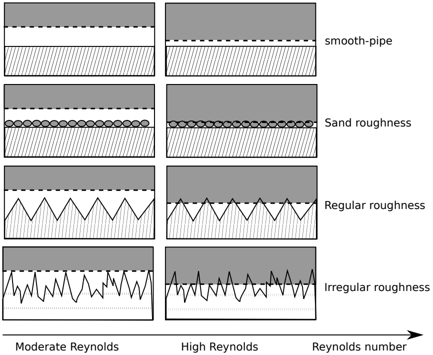
- Noor Afzal's (see 2007 [11]) universal relation $Re \to Re_\Phi = Re/\Phi$ (Roughness Reynolds)
- Φ roughness scale (for $\epsilon = 0$, $\Phi = 1$)
- Power-law $Re \in [15 \cdot 10^3, 6 \cdot 10^5]$, $K_{fit} = 0{,}185, m_{fit} = -0{,}20$, $R_{squared} = 0{,}998$

$$f_D = K_{fit} \cdot Re_\phi^{m_{fit}} = \frac{K_{fit}}{\phi^{m_{fit}}} \cdot Re^{m_{fit}}$$

$\widehat{K}$ changes with the roughness scale Φ which depends on $\dot{V}$.







$$\hat{\varphi} = f_D \cdot \frac{L}{D} \cdot \frac{\bar{v}^2}{2} \cdot \rho$$

$$\widehat{\varphi} = \underbrace{\frac{K_{fit}}{\phi^{m_{fit}}} \cdot \left(\frac{4}{\nu \pi D}\right)^{m_{fit}} \cdot \left(\frac{8\rho L}{\pi^2 D^5}\right) \cdot \dot{V}_T^{(2+m_{fit})}}_{\widehat{K}} \cdot \underbrace{\psi^{(2+m_{fit})}}_{\psi^m}$$

$$\widehat{\varphi} = \widehat{K} \cdot \psi^m$$

Smooth conduits:

- If $\epsilon = 0$ (smooth) then $\Phi = 1$ and $\widehat{K} > 0$ and constant.
- Geometry: $\{L, D\}$ are *external* parameters which *control* $\widehat{K}$.

For completely smooth conduits networks, the energy dissipation minimisation would give directly the steady-state flow distribution

What occurs, if m_{fit} is not is not used? and we take $m = 2$

$$\widehat{\varphi} = \underbrace{f_D \cdot \left(\frac{8\rho L}{\pi^2 D^5}\right) \cdot \dot{V}_T^{(2)}}_{\widehat{K}} \cdot \underbrace{\psi^{(2)}}_{\psi^m} = \widehat{K}(\psi) \cdot \psi^2$$

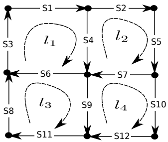
$$F(\psi_1, \ldots, \psi_{nsect}) = (F \circ g)(x_1, \ldots, x_n) =$$
$$= \sum_{j=1}^{nsect} \widehat{K}_j \cdot \psi_j^{(m+1)} =$$
$$= \sum_{j=1}^{nsect} \widehat{K}_j \cdot \psi_j^{((2+m_{fit})+1)} =$$
$$= \sum_{j=1}^{nsect} \widehat{K}_j(\psi_j) \cdot \psi_j^{(2+1)} > 0$$

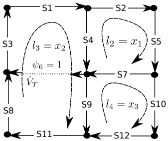
The minimisation problem turns into a fixed-point one:
Incorrectly tuned.
Cannot be tuned. Rough conduits: If $\epsilon \neq 0$ then $\widehat{K}$ has another *internal* paremeter Φ which depends on $\dot{V}$ (or ψ). In practice it is not possible to control $\{L, D, \Phi\}$ in order to keep $\widehat{K}$ constant.

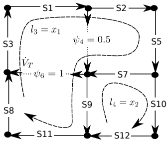
1. Start($i = 0$): initial guess flow distribution
   $g : \vec{x}^{(0)} \rightarrow \vec{\psi}^{(0)}$ compute $\vec{\widehat{K}}^{(0)} \in \mathbb{R}^{nsect}$.
2. Next ($i = i + 1$):
   Minimise the dissipation $\vec{x}^{(i)} = min_{\vec{x}} F$
3. $g : \vec{x}^{(i)} \rightarrow \vec{\psi}^{(i)}$ compute $\vec{\widehat{K}}^{(i)}$
4. $||\Delta \vec{\widehat{K}} = \widehat{K}^{(i)} - \widehat{K}^{(i-1)}|| < Tol?$,
   If not then store $\vec{\widehat{K}}^{(i)}$ and go to 2
   Otherwise we have got the solution $\vec{x}_{sol} = \vec{x}^{(i)}$.

General networks

Non tree-shaped networks







$$g : \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_5 \\ \psi_7 \\ \psi_8 \\ \psi_9 \\ \psi_{10} \\ \psi_{11} \\ \psi_{12} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} l_3 \\ l_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0{,}5 \\ 1 \\ 0{,}5 \\ 0{,}5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

For **general networks**. The key point is another map h;

$$h : \vec{\psi} \rightarrow (|\psi_1|, \dots)$$

The dissipation function becomes:

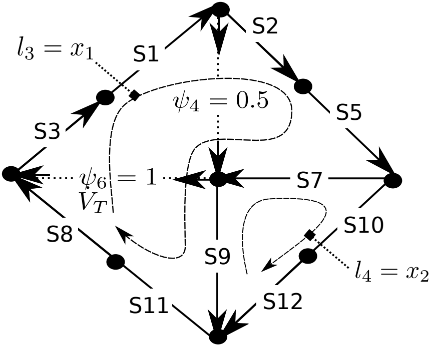
$$(F \circ h \circ g)(x_1, \dots, x_n) = \sum_{j=1}^{nsect} \widehat{K}_j \cdot |\psi_j|^{m+1} > 0$$

General form of the Energy Dissipation Function

$$\widehat{\varphi}_T = (F \circ h \circ g)(x_1, \dots, x_n)$$

Note: for instance, the previous equations for the tree-shaped example would be;

$$\text{sign}(\psi_2)\widehat{\varphi}_{S2} + \text{sign}(\psi_4)\widehat{\varphi}_{S4} - \text{sign}(\psi_3)\widehat{\varphi}_{S3} = 0$$
$$\text{sign}(\psi_2)\widehat{\varphi}_{S2} + \text{sign}(\psi_5)\widehat{\varphi}_{S5} - \text{sign}(\psi_3)\widehat{\varphi}_{S3} = 0$$



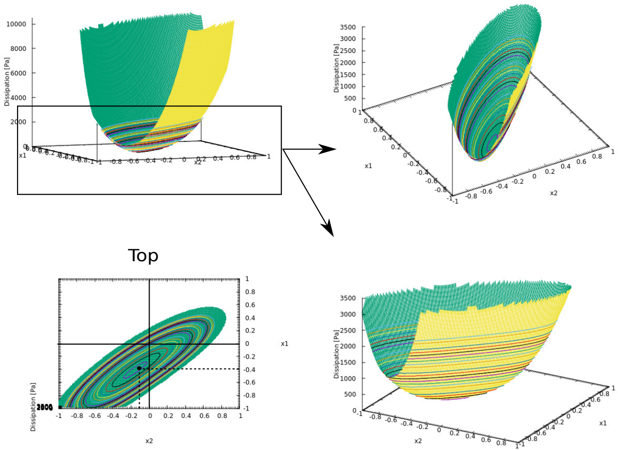
$\nu = 1{,}49 \cdot 10^{-5} [m^2/s]$
$\rho = 1{,}21 [kg/m^3]$

Sj: Section index *j*	$L[m]$
1, 2, 3, 5, 8, 10, 11, 12	10,0
7, 9	14,2
Sj: Section index *j*	$D[m]$
1, 3, 8, 11	0,6
2, 5, 10, 12	0,4
7, 9	0,3

Conduit roughness $\epsilon = 0{,}14$ (galvanized steel)

Minimisation with respect x_1 and x_2
Gradient-free minimisation: **Nelder-Mead's algorithm**

Visualization: Energy Dissipation Surface



CONCLUSIONS

- **New method** to compute the network steady-flow distribution (**neglecting interactions at branched junctions**).
- It can be applied to any type of network.
- **Gain insight** into the physical role of the power-law ($\widehat{\varphi} = \widehat{K} \cdot |\psi|^m$, particularly, K and m) as a measure of the energy dissipation.
- Near future work: extend the results to **branched junctions**. *Caveat: it cannot be done in straight forward manner.*

Thanks for your attention