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## **TEMPERATURE TUNABLE ACOUSTIC FILTER**

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### **Abstract**

We demonstrate theoretically that the periodic multilayer Epoxy/Glass supports acoustic frequency bands sensitive to the changes of temperature. Due to the strong variation of the sound velocity in the Epoxy, the width of the band gaps and the mid-gap frequencies descend when the temperature passes from 0 to 100°C. The transmittance spectrum shows that this system can be useful as a sound filter tunable with the temperature. The structural parameters were chosen to have temperature tuning in the ultrasound regime.

### **INTRODUCTION**

Acoustic – or *phononic* – crystals (PC's) are artificial periodic structures with an interesting property: they support a band structure for the sound vibrations. It means that acoustic energy can or cannot propagate through a PC, the cases when the frequency of the wave lays within an allowed band or in a forbidden band, respectively. In recent years much work has been performed in systems of one, [1-3]

two [4-6] and three [7,8] dimensional periodicity detailing the properties of these materials and proposing applications for the transducer and filter industry.

The theory of PC's was developed once the *photonic* crystals were introduced as useful systems for the control of light.[9-11] Up to date several mechanisms have been proposed for tuning the photonic bands. Tuning means variation of the frequency bands when the ambient conditions change without structural variation of the crystal. The change of temperature is one of such mechanisms.[12-13] In a similar manner, for practical applications, it is desirable a PC with tunable acoustic bands.

In this paper we demonstrate that the acoustic bands can be made tunable by changes of temperature (T-tuning). This effect requires that the elastic properties of at least one of the material constituents be temperature dependent. It is clear, if the bands depend on the material parameters and if these parameters vary with the temperature, so the bands are expected to change. We show this effect for a system of one dimensional periodicity, a solid superlattice.

As material constituents for the T-tunable superlattice we use epoxy resin and ordinary glass. Epoxy resins present a great variety of properties. They are currently employed in the acoustic industry as passive element between active transducer, for example. The important point for us is that the magnitude of its elastic modulus – thus the sound velocity and/or the mass density - depends on the temperature. Epoxy resins undergo the glass-rubber transition at the temperature  $T_g$ , which is a softening point at which the elastic modulus diminishes drastically [14]. We found ideal to our purposes the Epoxy of trade name L385:340. It has a  $T_g \sim 80^\circ\text{C}$  in the ultrasound regime. On the other hand from the great variety of glasses we selected the glass  $\text{Na}_2\text{O-TiO}_2\text{-SiO}_2$  with concentrations 30%-20%-50%.[15] This selection ensures high contrast between the epoxy and glass impedances,  $(Z_{\text{glass}}-Z_{\text{epoxy}})/Z_{\text{epoxy}} \times 100\% \sim 78\%$ , a requirement to obtain well defined acoustic gaps. The glass has about 1.4% of impedance variation in the range  $0 < T < 100^\circ\text{C}$ . Thus the T-tuning phenomenon that we shall describe for the superlattice Epoxy/Glass, results mainly from the epoxy properties.

## THEORY AND METHODS FOR CALCULATIONS

The superlattice is composed by alternated layers of Epoxy and Glass of thicknesses  $a$  and  $b$ , respectively. The layers are characterized by mass densities  $\rho_E$  and  $\rho_G$  and sound velocities  $c_E$  and  $c_G$ . In order to calculate the sonic bands we describe the periodic multilayer by a single periodic mass density  $\rho(z) = \rho(z + d)$  and also by a single sound velocity  $c_l(z) = c_l(z + d)$ , which of course can be written in terms of  $\rho_E$ ,  $\rho_G$  and  $c_E$ ,  $c_G$ , respectively. In these definitions  $d$  is the period of the structure – the size of the unit cell,  $d = a + b$ . We consider sound propagation only along the superlattice axis, the  $z$  axis. The displacement has also the  $z$  direction  $\vec{u} = u\hat{z}$ . The sonic modes are obtained from the wave equation

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial z} \left( \rho c_l^2 \frac{\partial u}{\partial t^2} \right) \quad (1)$$

which can be obtained from the theory of elasticity within the Cauchy approximation. For numerical solution of Eq. (1) we employ the method of plane wave expansion (PWEM). Thus we require the Fourier spectra of  $\rho$  and  $\rho c_l^2$ . Further, the field displacement must satisfy the Bloch theorem. We can write it as

$$\tilde{u}_k(z, t) = \sum_G u_G(k) \hat{z} \exp[i(k + G)z] \exp(-i\omega t) \quad (2)$$

where  $\vec{k} = k\hat{z}$  is the one dimensional Bloch vector and the vectors of the reciprocal lattice have the form  $\vec{G} = \frac{2\pi n}{d} \hat{z}$ . When Eq. (2) and the Fourier spectra above mentioned are introduced in Eq. (1) an infinite array of equations is obtained. We have found that a basis of 300 – 400 plane waves is enough to generate converged solutions. Note that this calculation does not use explicitly the boundary conditions for the field displacement. The boundary conditions are satisfied implicitly. Also, we shall present transmission spectra of sound traversing a superlattice sample. The sample is embedded in water. For this calculation no longer the PWEM can be used. Now we resolve the equations that result from the boundary conditions satisfied at each interface  $z_i$ ,

$$u^{(a)} \Big|_{z_i} = u^{(b)} \Big|_{z_i}, \quad (3)$$

$$\rho_a c_{al}^2 u_{zz}^a \Big|_{z_i} = \rho_b c_{bl}^2 u_{zz}^b \Big|_{z_i}, \quad (4)$$

the continuity of the displacement amplitude and the continuity of the normal stress, respectively. Explicitly,  $u_{zz}^{(a,b)} = \frac{\partial u_z^{(a,b)}}{\partial z}$ . Furthermore,  $k_z = \frac{\omega}{c_l}$ . For a single interface these conditions lead to the pair of equations

$$u_i + u_r = u_t, \quad (5)$$

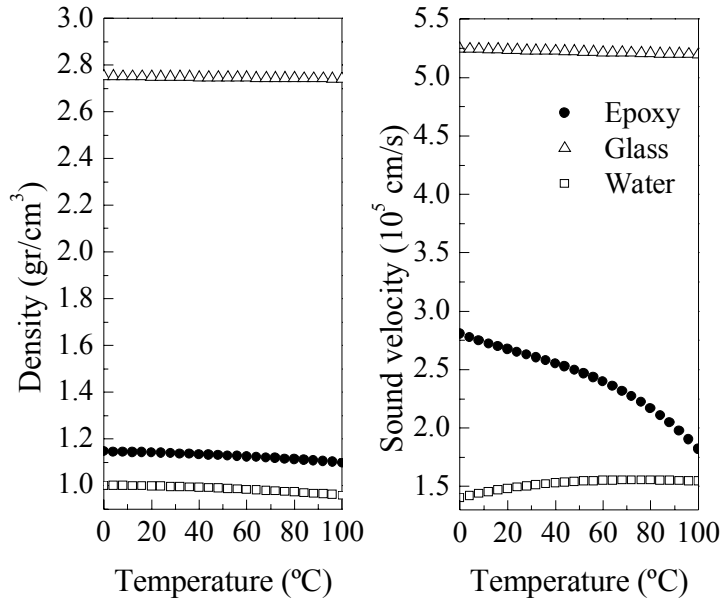
$$\rho_a c_{la} (u_i - u_r) = \rho_b c_{lb} u_t. \quad (6)$$

that involve the incident  $u_i$ , reflected  $u_r$  and transmitted  $u_t$  sound amplitudes. Now, for a sample of  $n$  cells, it is straightforward to obtain the system of  $2(2n+1)$  equations from which one can calculate numerically the spectra of transmission and reflection. Details of this point will be published elsewhere.

## EPOXY, GLASS AND H<sub>2</sub>O T-DEPENDENT PROPERTIES

As we mentioned above the Epoxy L385:340 could be an excellent material to construct PC's with temperature dependent sonic bands. This material suffers a drastic change of acoustic impedance  $Z = \rho c$  at the temperature  $T_g$ . Although  $T_g$  depends on the frequency, it remains between 80 and 100°C for all the range from the ultrasound until the infrasound. Due that the mass density of this material changes slightly within the range  $80 < T < 100^\circ\text{C}$ , the drastic change of the acoustic impedance affects the sound velocity mainly. Figure 1 shows for the Epoxy a strong variation of sound velocity. The density is almost unaffected. The curves were obtained directly from the experimental results appeared in Reference [14]. Figure 1 also shows that the Glass Na<sub>2</sub>O-TiO<sub>2</sub>-SiO<sub>2</sub> is insensitive to the temperature variation. Both density and sound velocity of the glass are practically constant in the range  $0 < T < 100^\circ\text{C}$ . It is easy to arrive to this conclusion. The density follows the function  $\rho(T) = \rho_0 [1 - 3\alpha(T - 25^\circ\text{C})]$ , [14] where  $\rho_0 = 2.749\text{gr/cm}^3$  and  $\alpha = 131 \times 10^{-7} \text{ }^\circ\text{C}^{-1}$ . Then, the speed of sound is  $c_l = \sqrt{\frac{E}{\rho}}$  with  $E = 3K(1 - 2\sigma)$ .  $K$  and  $\sigma$  satisfy the relations  $dK/dT = -0.0081\text{GPa} \times ^\circ\text{C}^{-1}$  and  $d\sigma/dT = 1.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ . At  $T = 25^\circ\text{C}$ , the bulk modulus is  $K = 50\text{ GPa}$  and the Poisson ratio is  $\sigma = 0.249$ .

Figure 1 also presents curves for Water which can be easily found in the literature.

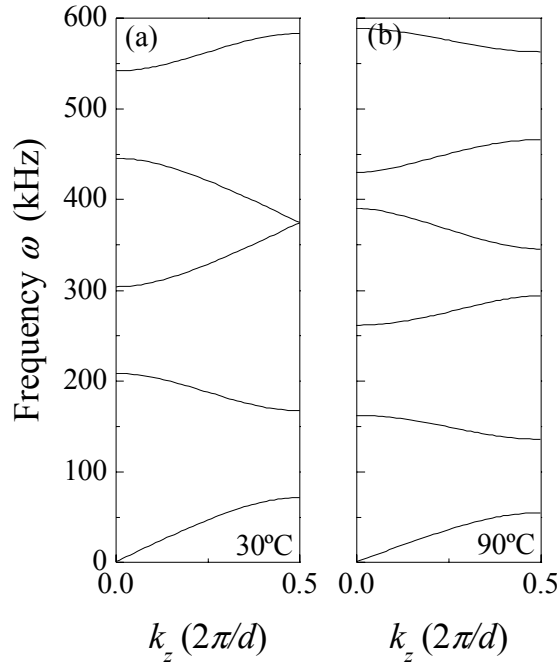


*Figure 1 - Dependence on temperature of material parameters of the superlattice constituents. The surrounded medium (Water) is also included. While the sound velocity of the Epoxy descends about 35% when the temperature varies from 0 to 100 °C, all the other parameters remain practically constant.*

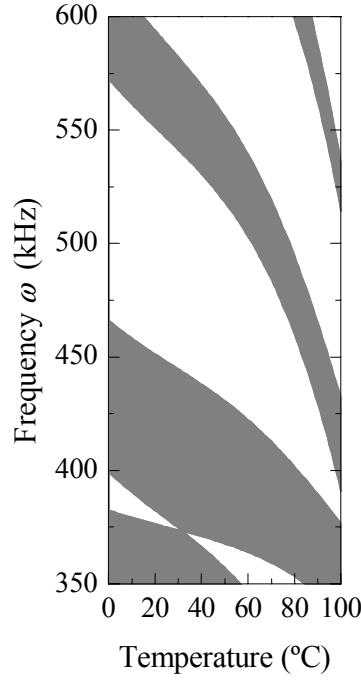
## T-TUNING OF THE FREQUENCY BANDS

Figure 2 shows the drastic effect on the band structure of our 1D PC when the temperature passes from 30 to 90°C. For the unit cell we have chosen  $d = 1.4$  cm ( $a = b = 7.0$  mm). Note that for 90°C two additional gaps open up and all the bands descend to lower frequencies. Sonic energy of frequency 150 kHz can not propagate at 30°C but at 90°C there exist an oscillatory mode allowing its transmission. On the other hand, for waves of 410 kHz the oscillatory mode at 30°C disappears at 90°C. The most important effects occur above the third and fourth bands. The degenerate point at the limit of the zone disappears due to the increase of the temperature. Further, the third gap, the gap with mid-frequency about 500kHz, descends almost 100 kHz shrinking its size 60%.

The behavior of the allowed and forbidden bands through all the range of temperatures  $0 < T < 100^\circ\text{C}$  is shown in Fig. 3 for the most interesting frequency region. Note that the lowest gap closes and opens up again as consequence of the temperature variation. The T-tuning gives place to a degenerated solution (the point of cross) at the frequency and temperature about 375 kHz and 31°C, respectively. The T-tuning of the second gap in Fig. 3 is representative of the strong effects caused by the changes of temperature. The original gap width ( $\sim 110$  kHz) shrinks almost 90% and the mid-gap frequency falls from 520 kHz to 384 kHz.



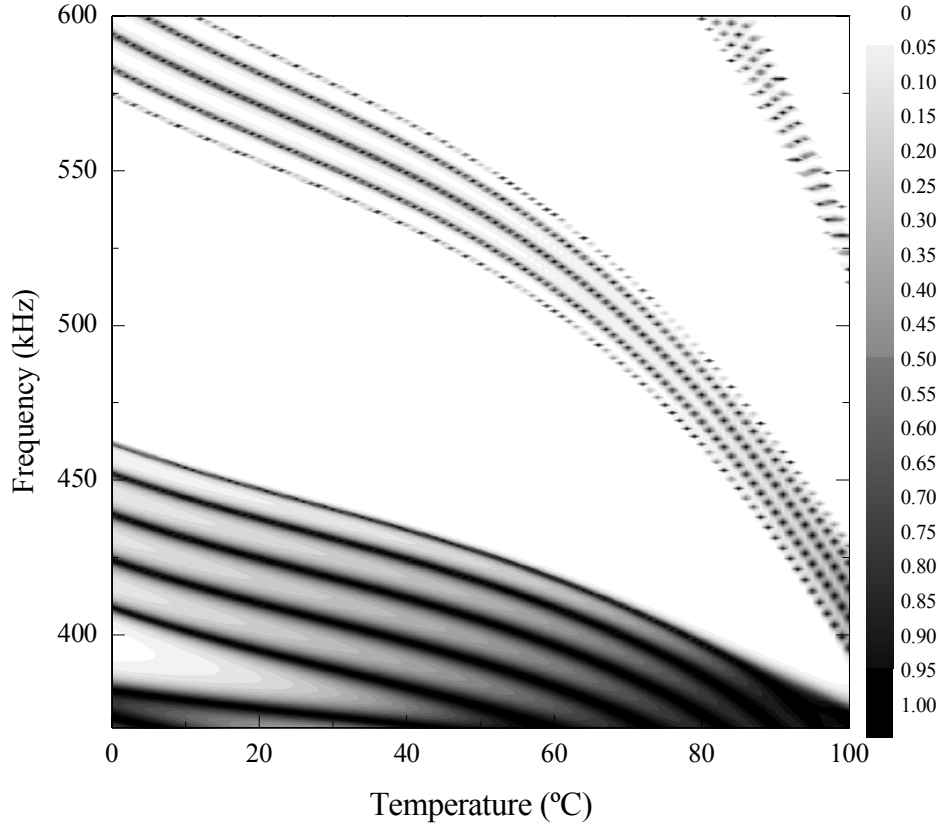
*Figure 2 - The acoustic band structure of the Epoxy/Glass superlattice. a) At 30 °C five bands appear within the scale of the figure. b) At 90 °C there exist six bands. The increasing of temperature shrinks the gaps by lowering the allowed bands.*



*Figure 3 -  $T$ -dependence of sonic bands. The temperature increasing produces a closing of the first forbidden band and a drastic falling of the other gaps.*

### **T-TUNABLE FILTERING**

We have calculated sound transmission for the system Water-Superlattice-Water. The multilayer contains six unit cells Epoxy/Glass. In order to have a symmetric sample we added a layer of Epoxy next to the last cell. Figure 4 shows the transmittance spectrum for normal incident waves. First that all we observe complete agreement of this spectrum with the band structure of Fig. 3; as is expected the transmission peaks lie within the frequency regions defined by the allowed bands of the infinite structure. The plots in Fig. 4 demonstrate that sound transmission through a finite superlattice embedded in water, all in thermal equilibrium, modifies as the temperature of equilibrium changes. The system is  $T$ -tunable. For example, sound of frequency 500 kHz will pass only when  $65 < T < 80^\circ\text{C}$ , approximately. For temperatures above or below to this range, sound of 500 kHz will be completely reflected.



*Figure 4 - Sound transmittance as function of temperature and frequency. The scale of color tones at the right means that dark zones correspond to peaks of transmission.*

It is very interesting the transference of modes between the lower two bands in Fig. 4. The connecting curve between the two bands becomes shorter and shorter as the number of cells increases in the sample. With infinite number of cells, the bands reach the nominal widths and the curve of connection drops to a point.

## CONCLUSIONS

We have demonstrated that the superlattice Epoxy/Glass supports T-tunable acoustic bands. In general by increasing the temperature the allowed and forbidden acoustic bands take lower frequency values. We found variations as higher as 25% of the mid-frequency gaps and 90% of the width gaps when the temperature changes from 0 to 100°C. Furthermore the continue variation of temperature can close and reopen a gap. We also calculate the spectra of transmission. The results show that this superlattice can be useful as a sound T-tunable filter for applications in the ultrasound regime.

We hope that this prediction stimulates experimental work for its practical demonstration.

## ACKNOWLEDGEMENT

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