Experimental evidence of anisotropic behaviour of dissipative photonic crystals in the homogenization limit

Jose Sánchez-Dehesa[1], Jorge Carbonell[1], Francisco Cervera[1], Lyudmila Gumen[2], Jesús Arriaga[3], and Arkadii Krokhin[4]

[1]Wave phenomena Group, Dep. of Electronic Engineering, Universidad Politécnica de Valecia C/ Camino de vera s.n., ES-46022, Valencia, Spain
Fax: + 3–963879609; email: jsdehesa@upvnet.upv.es
[2] Universidad Popular Autónoma del Estado de Puebla, Puebla, 72160, Mexico
[3] Instituto de Física, Universidad Autónoma de Puebla, Puebla, 72570, Mexico
[4] University of North Texas, Denton, TX 76203, USA

Abstract

We study the effects of anisotropy in the dielectric response of photonic crystals in the low-frequency limit. Both, real and imaginary part of the effective dielectric constant of Alumina and FR4 rods arranged in rectangular lattice have been measured in the microwave region. Experimental results for the E – and H – polarizations exhibit good agreement with our theoretical predictions obtained through the plane-wave expansion method as well as with the results of numerical simulations. It is found that the response is isotropic for the E – polarized mode, while the dielectric permittivity for the H – polarization depends on the direction of propagation.

1. Introduction

Photonic crystals (PC) have been theoretically studied in the long-wavelength limit (homogenization) because of their possible application as new materials for optical components [1]. Explicit formulas for the effective dielectric constants have been derived in Refs. [2,3]. They give the principal values of the dielectric tensor in terms of dielectric constants of the constituents ε_a and ε_b, and direction of propagation **n** = **k**/k. The PC constituents were assumed to be lossless, i.e. the values of ε_a and ε_b are real. However, finite dissipation leads to small imaginary part of the effective dielectric constant. Here we give analytic formulas for the imaginary part of the effective dielectric constant of a PC with anisotropic unit cell and compare the theory with experimental results obtained for microwave transmission through a periodic array of Alumina and FR4 rods.

2. Effective dielectric constant

We consider a PC of parallel rods (material a) imbedded in a matrix (material b) and calculate its dielectric response in the low-frequency limit $k, \omega \to 0$. We assume dissipative material for the rods and for the matrix with dielectric permittivities $\varepsilon_a = \varepsilon_a' + i\varepsilon_a''$ and $\varepsilon_b = \varepsilon_b' + i\varepsilon_b''$. In the limit of low dissipation $\varepsilon_a'' \ll \varepsilon_a'$, $\varepsilon_b'' \ll \varepsilon_b'$ we apply the Fourier expansion method [3] and for the E-polarized mode obtain the following result:

$$\frac{\varepsilon_E''}{\varepsilon_E'} = \frac{\eta_a'' - \eta_b''}{\eta_a' - \eta_b'} - \frac{\eta_a''\eta_b' - \eta_b''\eta_a'}{\eta_a' - \eta_b'} \varepsilon_E' \left[1 + \bar{\eta}^2 \left(\overline{\varepsilon^2} - \bar{\varepsilon}^2 \right)\right]. \tag{1}$$

Here η labels the inverse dielectric constant: $\eta_{a,b}' = 1/\varepsilon_{a,b}'$, $\eta_{a,b}'' = \varepsilon_{a,b}''/(\varepsilon_{a,b}')^2$. Bar over a symbol means average over the unit cell, e.g., $\bar{\eta} = f/\varepsilon_a' + (1-f)/\varepsilon_b'$, where f is the filling fraction of the component a. The real part ε_E' coincides with the average dielectric constant, $\varepsilon_E' = \bar{\varepsilon} = f\varepsilon_a' + (1-f)\varepsilon_b'$. For the E-mode ε_E' and ε_E'' are independent on the direction of propagation **n**. This symmetry comes from the fact that the lines of the electric field do not cross the interfaces of the cylinders. Unlike this, for the H-mode there is a refraction of the lines at the interface, therefore $\varepsilon_H(\mathbf{n})$=$\varepsilon_H'(\mathbf{n})$+ $i\varepsilon_H'(\mathbf{n})$ in general case changes with **n**.

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$$\frac{\varepsilon''_H}{\varepsilon'_H} = \frac{\eta''_a - \eta''_b}{\eta'_a - \eta'_b} - \frac{\eta''_a \eta'_b - \eta''_b \eta'_a}{\eta'_a - \eta'_b} \boldsymbol{\varepsilon}'_{\boldsymbol{H}}(\boldsymbol{n})(1 + n_i n_k B_{ik}), \quad i,k = x,y. \tag{2}$$

Here the real part $\boldsymbol{\varepsilon}'_{\boldsymbol{H}}(\boldsymbol{n})$ was calculated in Refs. [2,3], the tensor B_{ik} is given by

$$B_{ik} = \sum_{\mathbf{G}} Z_i(\mathbf{G}) Z_k(\mathbf{G}), \quad Z_i(\mathbf{G}) = \sum_{\mathbf{G}'} G'_i \eta(\mathbf{G}') \left[\mathbf{G} \cdot \mathbf{G}' \eta(\mathbf{G} - \mathbf{G}')\right]^{-1}, \tag{3}$$

and $\eta(\mathbf{G})$ is the Fourier component of the periodic function $1/\varepsilon'(\boldsymbol{r})$. The summations in eq. (3) run over all reciprocal lattice vectors $\mathbf{G}$ and $\mathbf{G}'$ and $[\cdots]^{-1}$ implies matrix inversion.

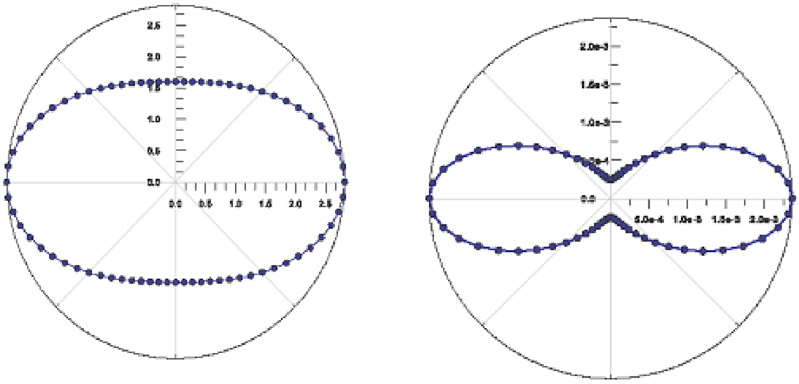

Fig. 1 Calculated from eqs. (2) and (3) angular dependence of the real (left) and imaginary (right) part of $\varepsilon_H(\mathbf{n})$ for rectangular lattice of Si cylinders ($\varepsilon'_a = 13.49$, $\varepsilon''_a = 0.0367$) with $f = 0.35$.

The real and imaginary part of $\varepsilon_H(\mathbf{n})$t exhibit anisotropy. In particular, for rectangular lattice with side ratio 1:2 the angular dependence of the real and imaginary part of ε_H is shown in Fig. 1. The real part (left panel) is an ellipse in agreement with Fresnel ellipsoid known from crystal optics. There is no such general result for the imaginary part (right panel), which turns out to be not a smooth function, exhibiting stronger anisotropy than that for the real part.

2. Experimental setup

Experimental demonstration of the previously introduced concepts of isotropy or anisotropy of the effective dielectric constant has been performed by means of microwave measurement. Two lattices made of alumina, $\varepsilon_a = 9.4(1 + i0.006)$, or FR4, $\varepsilon_a = 4.4(1 + i0.02)$, rods with diameter 4.2 and 3.2 mm, respectively, have been fabricated. The rectangular unit cells for these lattices are of 5×10 mm and 5×10 mm, leading to the filling fractions $f_{al} = 0.264$ and $f_{FR} = 0.254$. For the operation frequencies within 2–7 GHz the conditions of long-wavelength limit are well satisfied. The total extension of the samples in the propagation direction (≈ 10 cm) exceeds much the wavelength, i.e. the "bulk" approximation is valid. Two broadband horn antennas, covering the whole measured frequency range are used to transmit and receive plane waves with vertical polarization. The distance between the antennal varies from 55 to 125 cm that guarantees a detectable signal within far field zone.

3. Effective refractive index

The real part of the index of refraction is obtained from a measurement of the phase delay $\Delta\varphi$ in two samples with lengths l_1 and l_2, i.e. $n(\omega) = \sqrt{\varepsilon_{eff}} = -c\Delta\varphi/[\omega(l_2 - l_1)]$, where c is speed of light. For each polarization the data were collected for two directions of wave propagation – along the long side (0º incidence) and along the short side (90º incidence) of the rectangular unit cell. Results for the PC of the material FR4 are shown in Figs. 2 and 3. For the *E*-polarization we obtained two equal values $n_E^{(0)} = n_E^{(90)} = 1.33$ at 2GHz. This is in a reasonable agreement with the theory, $n_E = \sqrt{f_{FR}\varepsilon'_a + (1 - f_{FR})} = 1.365$. For the *H*-polarization two distinct values have been obtained, $n_H^{(0)} = 1.28$ and $n_H^{(90)} = 1.13$. This is also close to the corresponding theoretical results, $n_H^{(0)} = 1.24$ and $n_H^{(90)} = 1.15$, based on Refs. [2,3].

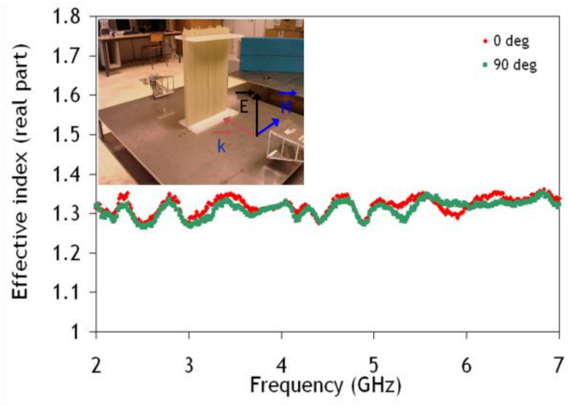

Fig. 2.- Dispersion of the effective refractive index measured for the PC of material FR4 with lengths ℓ_1=100 mm and ℓ_2=52 mm. *E*-polarized mode.

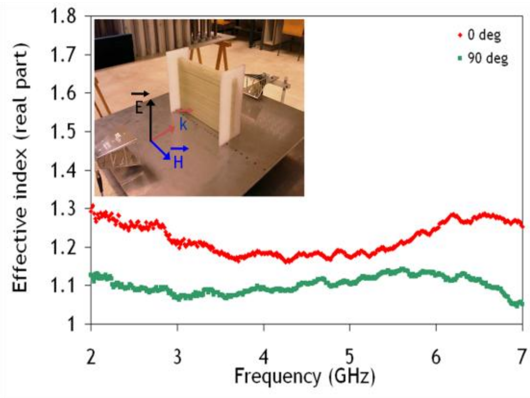

Fig. 3.- The same but for the *H*-polarized mode.

4. Homogenization of losses

Transmission and Reflection measurements have been performed to check the anisotropy of the imaginary part of the effective dielectric constant. Results shown in Figs. 4 and 5, demonstrate isotropic losses for the *E*-polarization and anisotropy for the *H*-polarization. Numerical simulations based on a finite-element method (COMSOL multiphysics) are also depicted for comparison purposes.

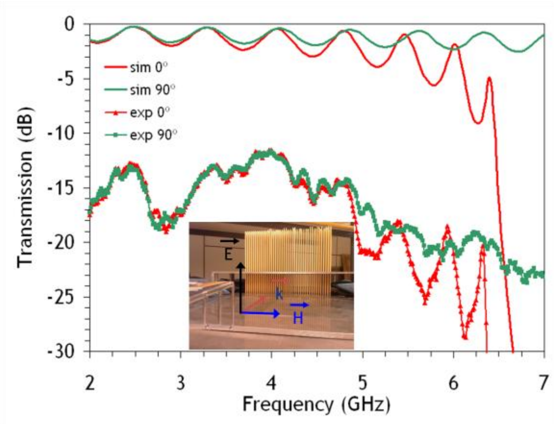

Fig. 4.- Measured (symbols) and simulated (lines) transmission through Al PC for the *E*-mode.

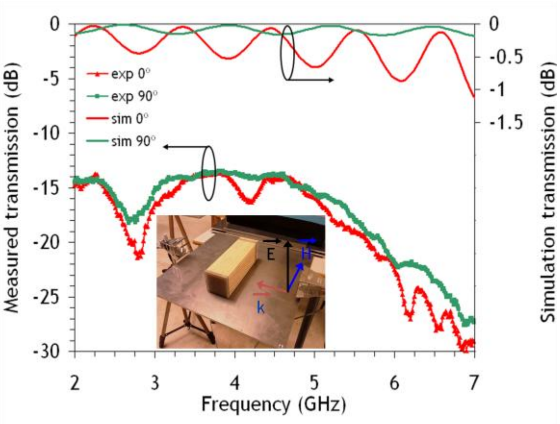

Fig. 5 – The same but for the *H*-mode.

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