

MATEMÁTICAS II - 03/01/2012

Assignment 2 to be handwritten and delivered on **11th January**. Presentation in English is encouraged and neatness will be had taken into account.

(Actividad 2 para ser escrita a mano y entregada el **miércoles 11 de enero**. La presentación y claridad se tendrán en cuenta.

1.- Chapter 5 Exercise 6 of SPUPV 798, i.e.: Find  $\mathcal{L} \left[ \frac{1}{4\sqrt[3]{x^3}} + \frac{1}{2\sqrt[10]{x}} - \sqrt[20]{x^3} \right]$  by using the  $\Gamma$  function.

2.- Chapter 5 Ex. 18: Show that  $\mathcal{L} \left[ \frac{\cos ax - \cos bx}{x} \right] = \frac{1}{2} \ln \frac{s^2 + b^2}{s^2 + a^2}$

3.- Chapter 5 Ex. 36: Find  $\mathcal{L} [f(x)]$  where  $f$  is an antiperiodic function whose period is 4 and satisfies  $f(x) = 2 - x$ ,  $0 \leq x < 1$ ,  $f(x) = 0$ ,  $1 \leq x < 2$ .

4.- Ch.5 Ex. 40 (b): Find  $\mathcal{L}^{-1} \left[ \frac{2s+1}{s^2-2s+2} \right]$

5.- Ch.5 Ex. 40 (k): Find  $\mathcal{L}^{-1} \left[ \frac{3s^2+8s-1}{(s-3)(s+2)^2} \right]$

6.- Ch.5 Ex. 51: Find the output of a dynamic system in which there is a  $\delta(t)$  impulse input and null initial conditions by knowing that the output is  $y(t) = e^{-2t} \cos(3t + \frac{\pi}{4})$  if the input is  $e(t) = 3u(t)$ , i.e. three times the Unit Step function.

7.- Ch. 5 Ex. 53(b): Solve  $y'' + 4y' + 4y = t - tu(t-1) - e^{-2(t-1)}u(t-1)$ , with initial conditions  $y(0^+) = 1, y'(0^+) = 0$

8.- Ch. 5 Ex. 59: Solve

$$\left. \begin{aligned} ty'' - ty' - y &= 0 \\ y(0^+) &= 0, \quad y'(0^+) = 3 \end{aligned} \right\}$$

9.- Ch. 5 Ex. 61: Solve the IVP

$$y'' + y + \int_0^x e^{2(x-t)} y'(t) dt - \int_0^x (x-t)y''(t) dt = e^{2x}$$

with  $y(0^+) = 0, y'(0^+) = 0$ .

10.- Ch. 5 Ex. 67: Find the intensities  $i_1(t), i_2(t)$  that satisfy

$$\left. \begin{aligned} 200i_1 + 200i_2 + \frac{di_2}{dt} &= 50 \\ 5i_1 + 4i_2 + 1000 \int_0^t i_1(t) dt &= 1 \end{aligned} \right\} \quad i_1(0) = i_2(0) = 0.$$

11.- Notes EDP Ch. 1 Ex. 1: Solve

$$(D_x + D_y + 1)(D_x - D_y)z = 3x^2 + 2y - 4.$$

12.- Notes EDP: The temperature  $T$  in a frigorific chamber is given by

$$\frac{\partial T}{\partial t} = \frac{1}{4} \frac{\partial^2 T}{\partial x^2}, \quad t > 0, \quad 0 < x < 10.$$

The initial temperature is  $20^\circ$  and the temperature on two walls are fixed at  $0^\circ$ , i.e.

$$T(0, t) = 0, \quad T(10, t) = 0,$$

$$T(x, 0) = 20.$$

Find  $T$  in the centre of the fridge. What will be the value of  $T$  in the centre of the frigorific chamber at  $t = 2$ ?